Q1

10 Points

[New instruction: on *all* of these problems (Questions 1-5) you may use a calculator/compute for *arithmetic only*, i.e., simplifying expression which involve $+, \cdot, -, /$.]

Answer the following multiple-choice questions. Double-check your answers, since no partial score would be given for these questions.

Q1.1

2 Points

How many solutions does the following system have:

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 4z &= 3 \\ x + 4y + 5z &= 4 \end{aligned}$$

- ○ one
- ○ infinitely many

Select a question.

Q1.1

2 Points

How many solutions does the following system have:

$$
\begin{aligned}
x + 2y + 3z &= 1 \\
x + 3y + 4z &= 3 \\
x + 4y + 5z &= 4
\end{aligned}
$$

- ○ one
- ○ infinitely many
- ◉ none

Q1.2

2 Points

The partial fraction decomposition of $\frac{t^2 - 2}{(t+1)^2(t^2+1)}$ should be of the form

- ○ $\frac{At}{(t+1)^2} + \frac{Bt+C}{t^2+1}$
- ○ $\frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{C}{t^2+1}$
- ◉ $\frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{Ct+D}{t^2+1}$
- ○ $\frac{A}{(t+1)} + \frac{B}{(t+1)^2} + \frac{Ct}{t^2+1}$

Q1.3

2 Points

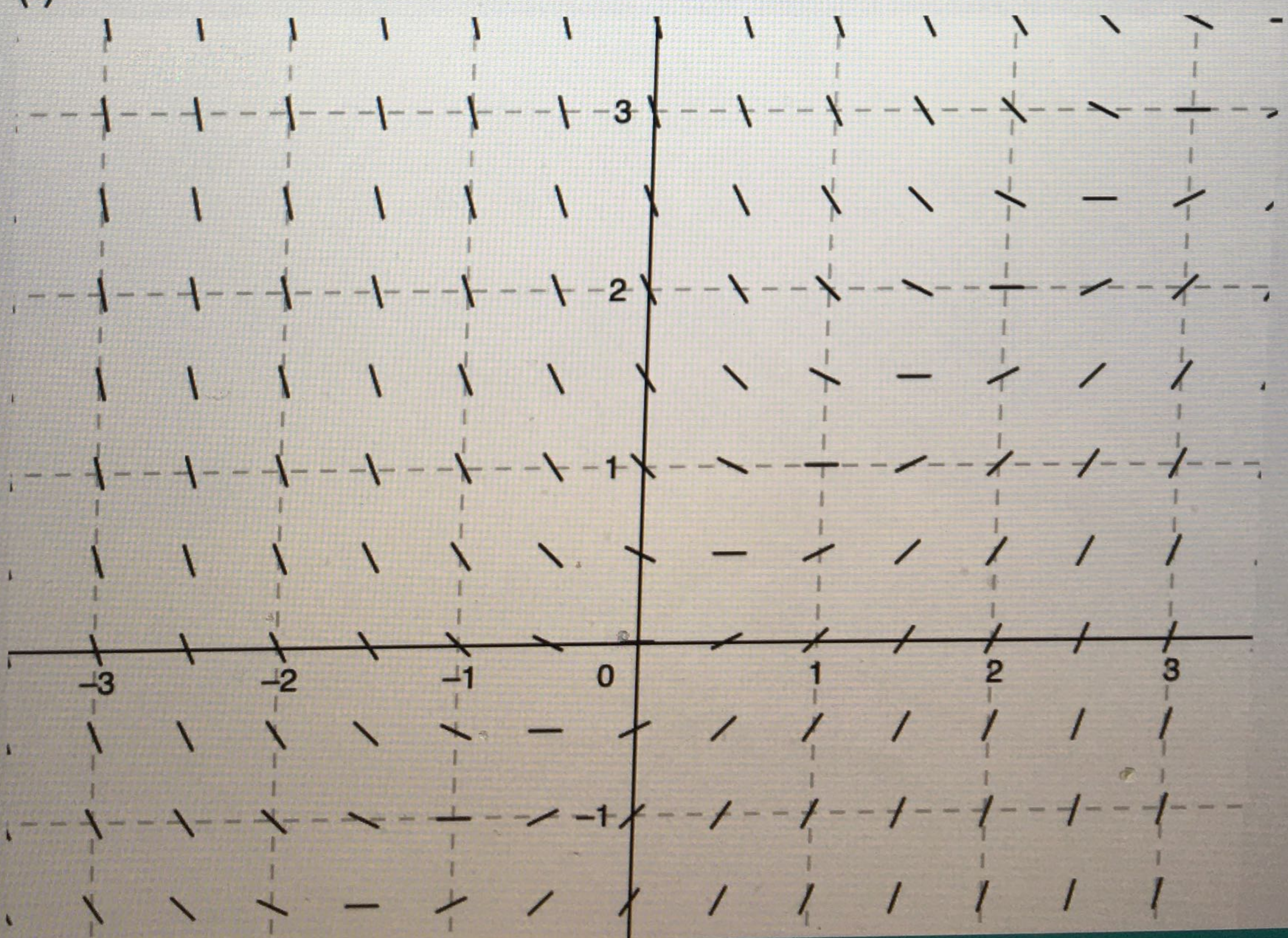
Match the following differential equations with their direction fields.

A: $y' = 2 - y$

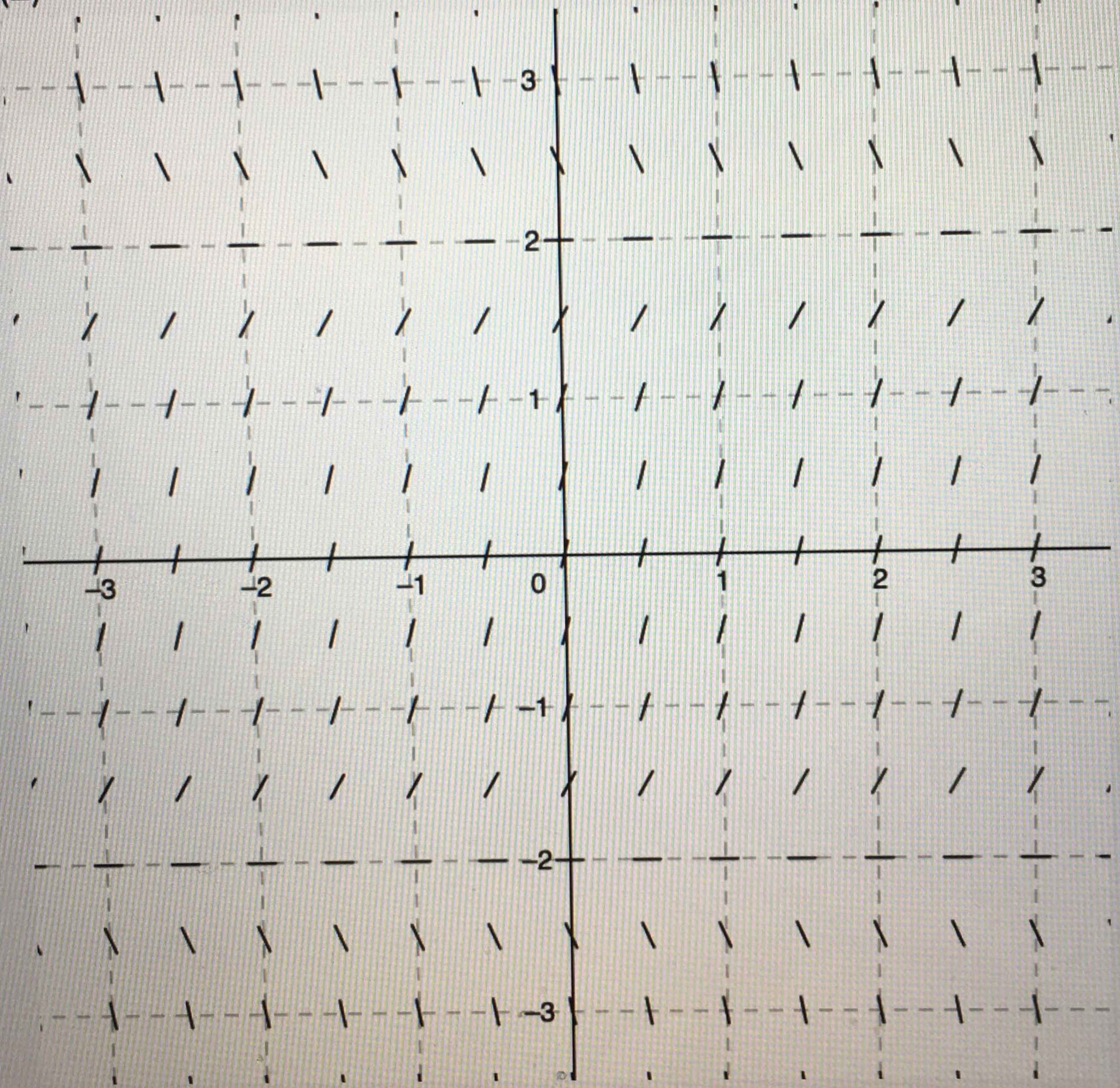
B: $y' = (y + 2)(2 - y)$.

C: $y' = t - y$

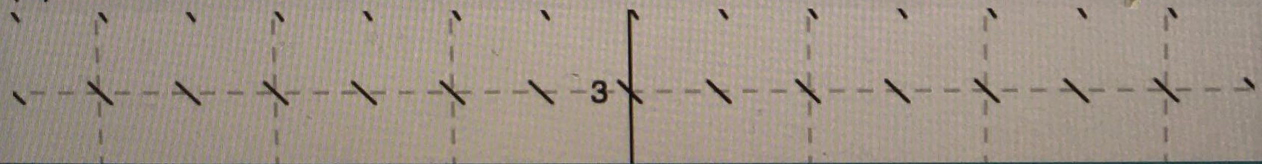
(1)



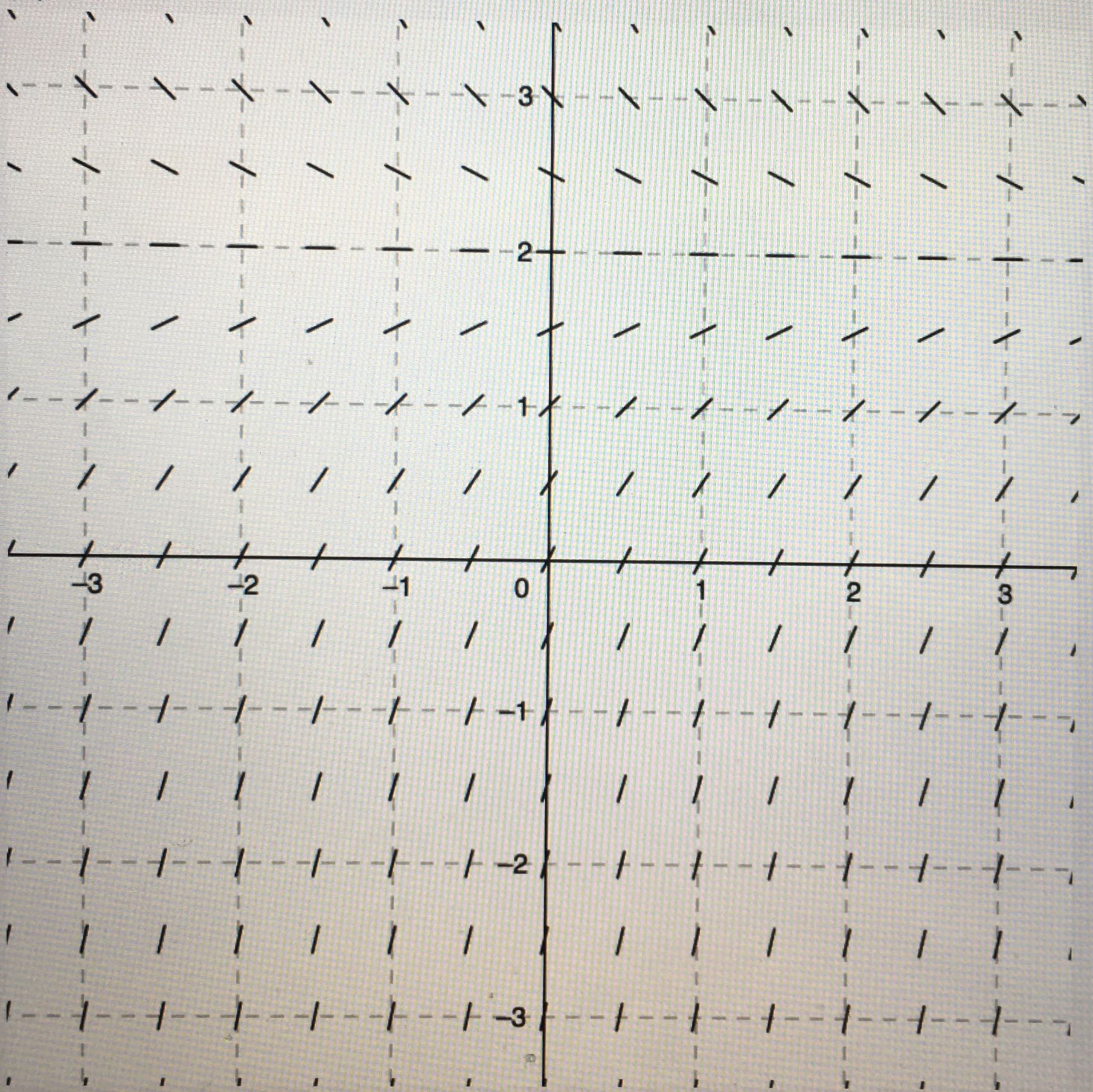
(2)



(3)

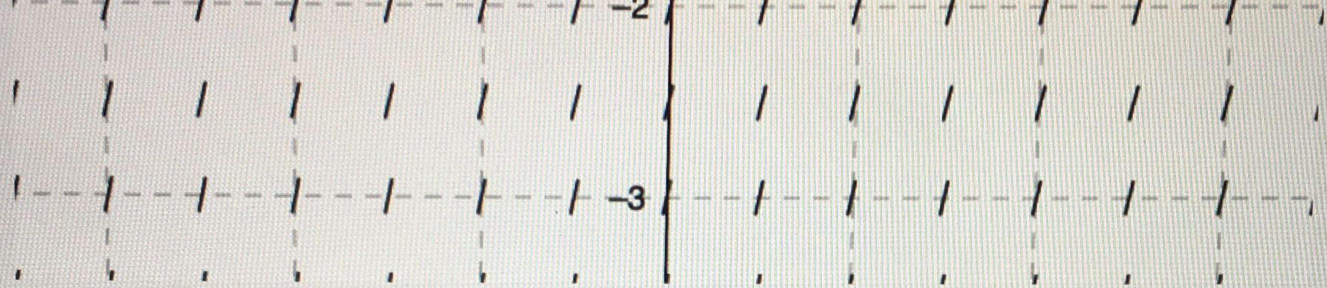


(3)



(1) is for

C



(1) is for

C

(2) is for

B

(3) is for

A

Q1.4

2 Points

Which of the following equations can be rearranged into separable equations?

Q1.4

2 Points

Which of the following equations can be rearranged into separable equations?

- [ ] $y' - y^2 = t$
- [x] $y' + yt^2 = t^2$
- [x] $ty' - e^t \sin(y) = e^t$
- [ ] $y' - \sin(t)y = t$

Q1.5

2 Points

Which of the following differential forms are exact?

- [x] $(y^2 - t^2)\, dt + (2ty - y)\, dy$
- [ ] $ye^t\, dt - e^t\, dy$

Q1.5

2 Points

Which of the following differential forms are exact?

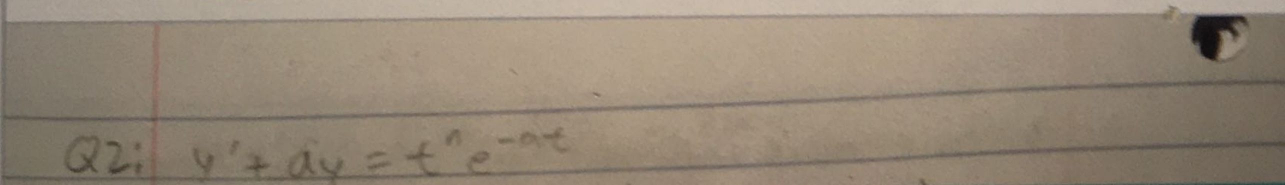
- [x] $(y^2 - t^2)\,dt + (2ty - y)\,dy$
- [ ] $ye^t\,dt - e^t\,dy$
- [x] $3\cos(3t - y)\,dt - \cos(3t - y)\,dy$
- [ ] $(2t^2y - 1)\,dt + t^3\,dy$

Q2

20 Points

Find the general solution for the following differential equation:
$y' + ay = t^n e^{-at}$, where $a \in \mathbb{R}$ and $n \in \mathbb{N}$.

▾ Mt1-p1.jpg Download

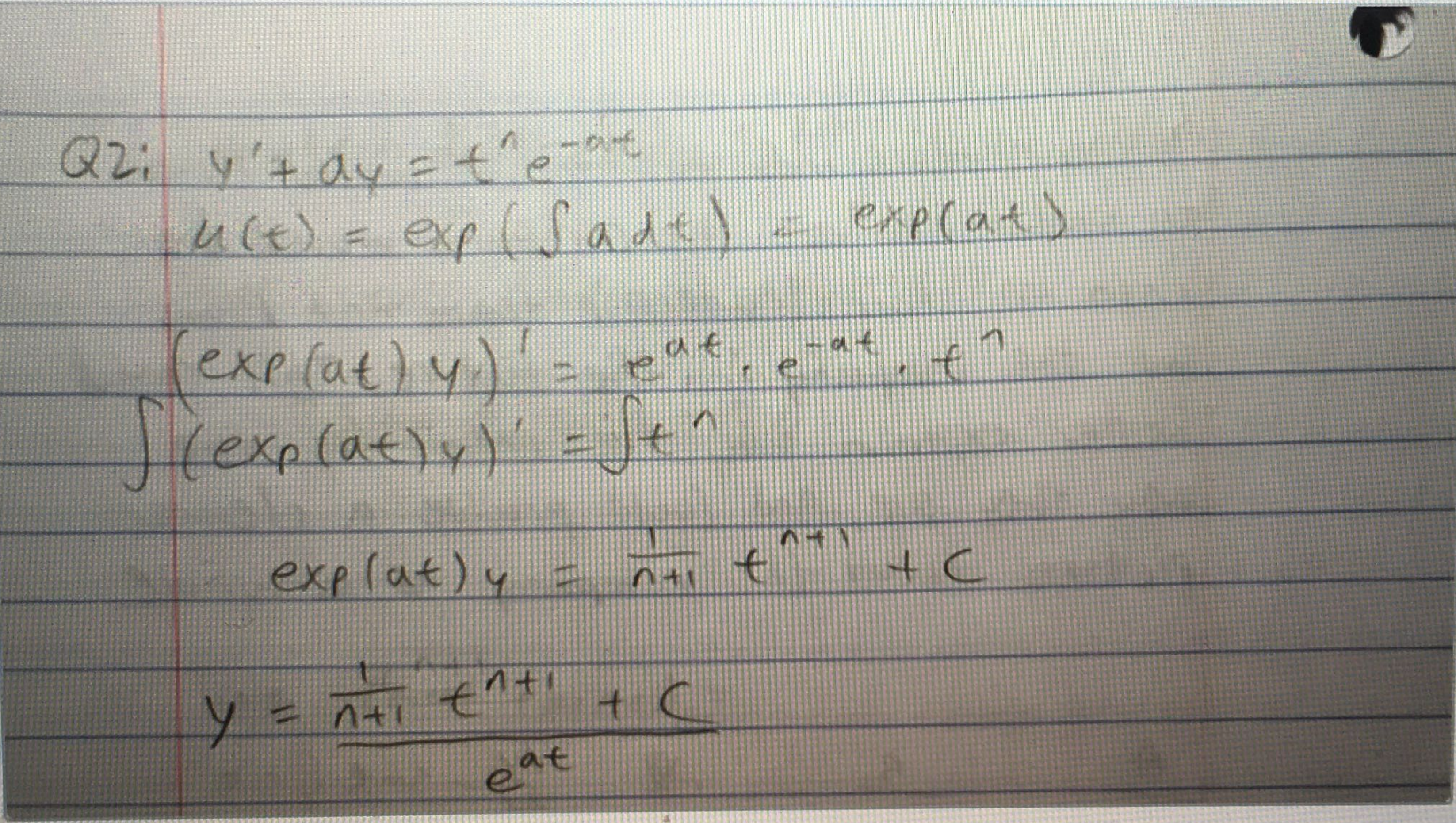

Q2: $y' + ay = t^n e^{-at}$

Q2

20 Points

Find the general solution for the following differential equation:
$y' + ay = t^n e^{-at}$, where $a \in \mathbb{R}$ and $n \in \mathbb{N}$.

▾ Mt1-p1.jpg   Download


Q2: $y' + ay = t^n e^{-at}$

$u(t) = \exp\left(\int a\,dt\right) = \exp(at)$

$(\exp(at)\,y)' = e^{at} \cdot e^{-at} \cdot t^n$

$\int (\exp(at)\,y)' = \int t^n$

$\exp(at)\,y = \frac{1}{n+1} t^{n+1} + C$

$y = \dfrac{\frac{1}{n+1} t^{n+1} + C}{e^{at}}$

Q3

25 Points

A tank contains 100 gallons of brine made by dissolving 80 lb of salt in water. Pure water runs into the tank at the rate of 4 gallons/minute, and the mixture, which is kept uniform by stirring, runs out at the same rate. Find the amount of salt in the tank at any time t. Find the concentration of salt in the tank at any time t

.

▾ Edit2-p1.jpg Download

Q3: Rate in: $\frac{4\text{ gal}}{\text{min}} \cdot \frac{0\text{ lbs}}{\text{gal}} = \frac{0\text{ lbs}}{\text{min}}$

Rate out: $\frac{4\text{ gal}}{\text{min}} \cdot \frac{S(t)}{100\text{ gal}} = \frac{4\,S(t)\text{ lbs}}{100\text{ min}}$

$S'(t) = -\frac{4\,S(t)}{100} = \frac{-S(t)}{25}$

$\frac{dS}{dt} = \frac{-S(t)}{25}$ $\quad \int \frac{1}{S(t)}\,dS = \int -\frac{1}{25}\,dt$

$= \ln(S(t)) = -\frac{1}{25}t + C$

$S(t) = e^{-1/25\,t} \cdot e^{C} = e^{-1/25\,t} \cdot C$

$S(0) = 80 = e^{0} \cdot C$ $\quad C = 80$

Salt at $t = 80e^{-1/25\,t}$

Conc at $t = \frac{80e^{-1/25\,t}}{100}$

Q4

20 Points

Solve the following initial value problem and determine the respective interval of existence:

$te^{t^2} + yy' = 0, \quad y(0) = 1.$

▾ Edits-p2.jpg

Download

Q4: $te^{t^2} + yy' = 0$, $y(0) = 1$

$-y\frac{dy}{dt} = te^{t^2}$ $\quad \int -y\,dy = \int te^{t^2}\,dt$

$\int -y\,dy = -\frac{1}{2}y^2$

$\int te^{t^2}\,dt$: $\quad u = t^2 \quad du = 2t\,dt$

$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u = \frac{1}{2}e^{t^2}$

$-\frac{1}{2}y^2 = \frac{1}{2}e^{t^2} + C$

$-\frac{1}{2}(1)^2 = \frac{1}{2}e^0 + C$, $\quad -1/2 = 1/2 + C$, $\quad C = -1$

$-\frac{1}{2}y^2 = \frac{1}{2}e^{t^2} - 1$

$y^2 = -e^{t^2} + 2$

$y = \sqrt{-e^{t^2} + 2}$

$-e^{t^2} + 2 \geq 0 \quad -e^{t^2} \geq -2$

$e^{t^2} \leq 2 \quad t^2 \ln e \leq \ln(2)$

Q5

25 Points

Check that the following differential form are exact and find the solution to the corresponding initial value problem:

$$\frac{y}{t+1}\,dt + (\ln(t+1) + 3y^2)\,dy = 0, \quad y(0) = 1.$$

▾ Mt1-p4.jpg Download

Q5: $\frac{y}{t+1}\,dt + (\ln(t+1) + 3y^2)\,dy = 0$

$P(y,t) = \frac{y}{t+1}$ $\quad Q(y,t) = \ln(t+1) + 3y^2$

$\frac{y}{t+1}\frac{\partial}{\partial y} = \frac{1}{t+1}$

$(\ln(t+1) + 3y^2)\frac{\partial}{\partial t} = \frac{1}{t+1}$

Since $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial t}$, the differential equation is exact.

$\int \frac{\partial F}{\partial t} = \int \frac{y}{t+1}$ $\quad F = \ln|t+1|\,y + h(y)$

$\int \frac{\partial F}{\partial y} = \int \ln(t+1) + 3y^2$ $\quad F = y^3 + \ln(t+1)\,y + g(t)$

$F = \ln|t+1|\,y + y^3 = C$

$\ln|0+1|\cdot(1) + (1)^3 = C$