Math 32B Exam 1

TOTAL POINTS

50 / 50

QUESTION 1

TF 8 pts

1.1 TF 2 / 2

✓ **- 0 pts** **True**

1.2 Yes/No Integrals 6 / 6

✓ **- 0 pts** **no no no no yes yes**

QUESTION 2

Worksheet Question 10 pts

2.1 Rectangular coords 4 / 4

✓ **- 0 pts** **Correct**

2.2 Polar coords 6 / 6

✓ **+ 6 pts** **Correct**

+ **1 pts** Correct bound
+ **2 pts** Correct bounds
+ **1 pts** Correct integrand (excluding Jacobian)
+ **2 pts** Jacobian
+ **1 pts** Correct final answer
- **1 pts** Minor Miscalculation/Incorrect final answer
+ **0 pts** incorrect or nothing shown

QUESTION 3

Non-linear transformation 10 pts

3.1 Picture 4 / 4

✓ **- 0 pts** **Correct (third picture)**

3.2 Integral 6 / 6

✓ **- 0 pts** **Correct**

QUESTION 4

Q4 10 pts

4.1 Sphere/Cone 4 / 4

✓ **- 0 pts** **Correct**

4.2 Volume integrals 6 / 6

✓ **- 0 pts** **Correct**

QUESTION 5

MC 12 pts

5.1 Spherical Plane 3 / 3

✓ **- 0 pts** **theta=pi/4**

5.2 Jacobian 3 / 3

✓ **- 0 pts** **2u^2+2v^2**

5.3 Cylindrical Plane 3 / 3

✓ **- 0 pts** **r=1/cos\theta**

5.4 Linear Map 3 / 3

✓ **- 0 pts** **(6u+2v, u+4v)**

Sign your name on the line below if you do **NOT** want your exam graded using GradeScope. Otherwise, keep it blank. If you sign here, we will grade your paper exam by hand and a) you will not get your exam back as quickly as everyone else, and b) you will not be able to keep a copy of your graded exam after you see it.

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- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	Total
Points:	8	10	10	10	12	50
Score:						

$\cos 2\phi = 1-2\sin^2\phi$

$2\sin^2\phi = \frac{1-\cos 2\phi}{2}$

$\int_1^4 \int_0^1 \sqrt{y}\,dx\,dy = \int_1^4 [\sqrt{y}\,x]_0^1\,dy = \int_1^4 \sqrt{y}\,dy = \left[\frac{2}{3}y^{\frac{3}{2}}\right]_1^4 = \left(\frac{2}{3}\times 8 - \frac{2}{3}\right) = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$

$\frac{13}{3}$

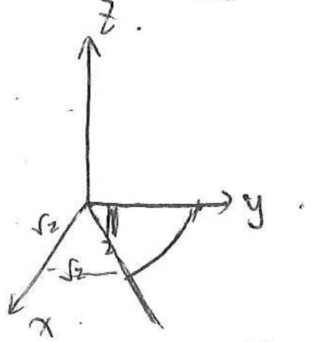
1. (8 points) (a) True or False? (circle one) $\displaystyle\int_1^4 \int_0^1 \sqrt{y}\,\sin(x^2y^2)\,dx\,dy \le 6$

True False

$\int_1^4 y^{\frac{1}{2}}\,dy$

$= \left[\frac{2}{3}y^{\frac{3}{2}}\right]_1^4 = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$

(b) Let D be the region in the positive octant ($x, y, z \ge 0$) enclosed by the sphere $x^2+y^2+z^2 = 4$ and the planes $z = 0$, $x = 0$, and $x = y$. For **each** integral below, circle "yes" or "no" depending on whether or not it equals $\displaystyle\iiint_D x\,dV$. $\iiint_S \rho\sin\phi\cos\theta\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$



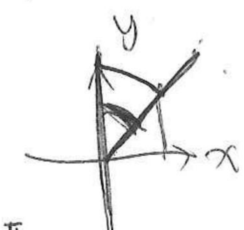
yes no $\displaystyle\int_0^{\pi/2}\int_0^{\pi/4}\int_0^2 \rho^3\cos\theta\sin^2\phi\,d\rho\,d\theta\,d\phi$

$\left(1-\frac{1}{\sqrt{2}}\right)\times[$

No!

yes no $\displaystyle\int_0^{\sqrt{2}}\int_{0 \to x}^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} x\,dz\,dy\,dx$

$\int \sin^2\phi\,d\phi$



yes no $\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\cos\theta\sin\phi\,d\rho\,d\theta\,d\phi$ $= \int \frac{1-\cos 2\phi}{2}\,d\phi$

$= \frac{1}{2}\left[\phi - \frac{1}{2}\sin 2\phi\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left(\frac{\pi}{2} - \right) = \frac{\pi}{4}$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2$,

$\theta \quad \phi \quad \rho$

yes no $\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2\cos\theta\sin\phi\,d\rho\,d\phi\,d\theta$

$\theta \quad \phi$

yes no $\displaystyle\int_0^2\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{4-r^2}} r^2\cos\theta\,dz\,d\theta\,dr$

~~$\cos\theta$~~

$[+\sin\theta]_0^{\frac{\pi}{4}} = +\sin\frac{\pi}{4} = +\frac{1}{\sqrt{2}}$

yes no $\displaystyle\int_0^{\pi/2}\int_0^2\int_{\pi/4}^{\pi/2} \rho^3\cos\theta\sin^2\phi\,d\theta\,d\rho\,d\phi$

$[\sin\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 1 - \sin\frac{\pi}{4} = 1 - \frac{1}{\sqrt{2}}$

$1\times[-\cos\phi]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = +\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

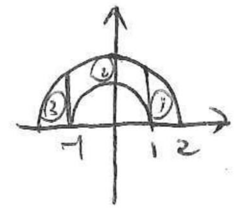
2. (10 points) Let R be the region in $\mathbb{R}^2$ which lies above the x-axis and between the circles of radius 1 and 2 centered at $(0, 0)$.

(a) Write the following integral as a sum of integrals in **rectangular coordinates:**

$$\iint_R 3y\, dA.$$

Do not evaluate these integrals. Box your answers.

$$\boxed{\int_1^2 \int_0^{\sqrt{4-x^2}} 3y\, dy\, dx + \int_{-1}^{1} \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} 3y\, dy\, dx + \int_{-2}^{-1} \int_0^{\sqrt{4-x^2}} 3y\, dy\, dx}$$



(b) Evaluate the integral in part (a) using **polar coordinates.** Box your answer.

$$\int_0^{\pi} \int_1^2 3 r^2 \sin\theta \, dr\, d\theta .$$

$$= 3 \int_0^{\pi} \sin\theta \, d\theta \int_1^2 r^2\, dr .$$

$$= 3 \left[-\cos\theta \right]_0^{\pi} \left[\frac{1}{3} r^3 \right]_1^2$$

$$= 3 (1+1) \left(\frac{8}{3} - \frac{1}{3} \right) .$$

$$= 3 \times 2 \times \frac{7}{3}$$

$$= \boxed{14}$$

$(1,v) = (1+v+v,\ 2v-1+2v)$ $= (2v+1,$

$x = u+v+uv$

$y = -u+2v+2uv$

$G(1,1) = (3,3)$

3. (10 points) Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be the non-linear transformation $G(1,0) = (1,-1)$

$u=1$

$$G(u,v) = (u + v + uv, -u + 2v + 2uv)\,.\quad G(0,1) = (1,2)$$

Let R be the unit square $[0,1] \times [0,1]$ in the uv-plane and let $D = G(R)$ in the xy-plane.

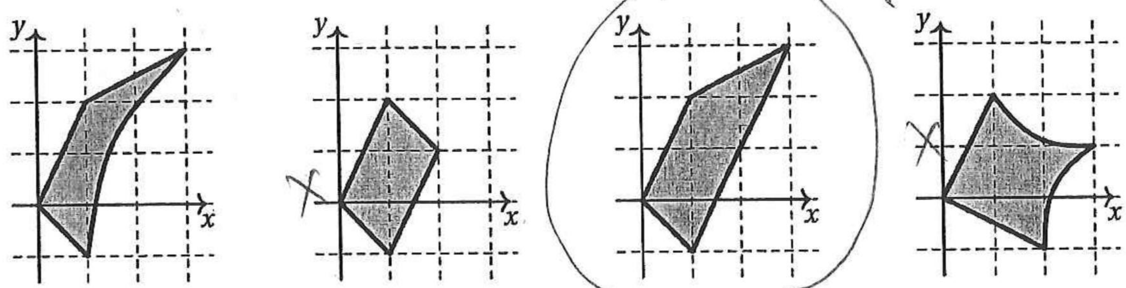
$2x-y=3u$ (a) Circle the picture of D below. The dashed grid consists of unit squares.

$u = \frac{2x-y}{3}$

$2x-y=3$

$y=2x-3$

$u=1$



(b) Find the limits and integrand of the integral below so that it equals

$$\iint_D \sqrt{x}\, dA$$

as an integral over the square R. Do not evaluate the integral. Show your work.

$$G: \begin{cases} x = u+v+uv \\ y = -u+2v+2uv \end{cases}$$

$$\mathrm{Jac}(G) = \det\begin{bmatrix} 1+v & 1+u \\ -1+2v & 2+2u \end{bmatrix} = (1+v)(2+2u) - (1+u)(2v-1)$$

$= 2+2u \ldots +1 \ldots +u$

$= 3+3u$

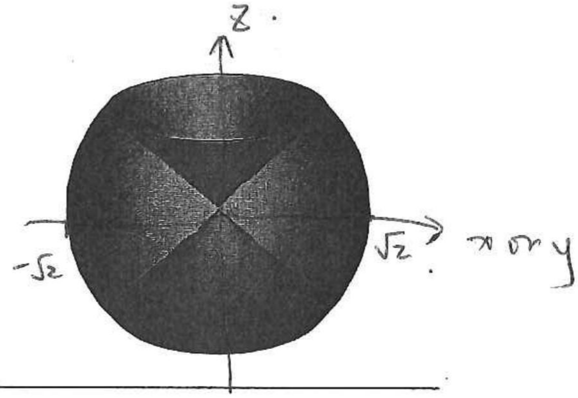
$\sqrt{x} = \sqrt{u+v+uv}$

u bound: $0 \le u \le 1$

v bound: $0 \le v \le 1$

$$\boxed{\iint_D \sqrt{x}\, dA = \int_0^1 \int_0^1 \sqrt{u+v+uv}\,(3+3u)\, du\, dv}$$

4. (10 points) (a) In **spherical coordinates**, describe the region outside the cone $x^2 + y^2 = z^2$ and inside the sphere $x^2 + y^2 + z^2 = 2$ (shown below – the sphere is translucent so you can see the cone inside).



$0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \phi \le \frac{3}{4}\pi$

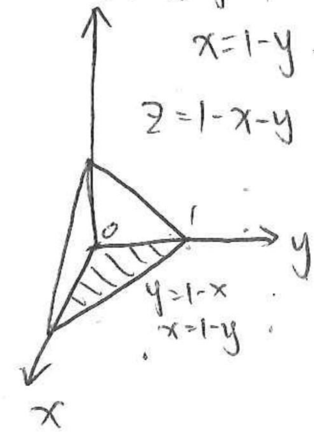
$0 \le \rho \le \sqrt{2}$

(b) Fill in the limits and integrand of the double and triple integrals below so that they both equal the volume of the region in the first octant ($x, y, z \ge 0$) below the plane $x+y+z=1$. Be sure to follow the provided order of integration.

$x+y=1$

$x=1-y$

$z=1-x-y$



$$\text{Vol} = \int_0^1 \int_0^{1-y} 1-x-y \; dx\, dy$$

$x+z=1$

$z=1-x$

$$\text{Vol} = \int_0^1 \int_0^{1-z} \int_0^{\cancel{1-x-y}\ 1-x-z} 1 \; dy\, dx\, dz$$

$\cos\theta = \sin\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

5. (12 points) Multiple choice. Circle the correct answer.

(a) In spherical coordinates the plane $y = x$ can be written as

$$\rho = \frac{1}{\cos\phi} \qquad \phi = \frac{\pi}{3} \qquad \rho = 1 \qquad \theta = \frac{\pi}{4} \qquad \rho = \frac{1}{\sin\phi}.$$

(b) The Jacobian of the map $G(u, v) = (u^2 - v^2, uv)$ is

$$2u^2 + 2v^2 \qquad 2u^2 - 2v^2 \qquad 4uv \qquad 2u + 2v \qquad -4uv$$

$$\det\begin{bmatrix} 2u & -2v \\ v & u \end{bmatrix} = 2u^2 + 2v^2 .$$

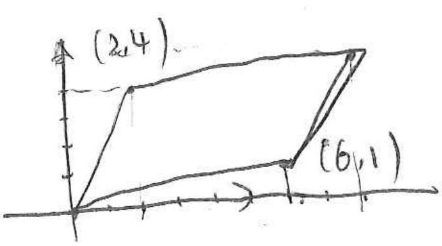
(c) In cylindrical coordinates the plane $x = 1$ can be written as

$$r = \frac{1}{\cos\theta} \qquad \theta = \frac{\pi}{3} \qquad r = 1 \qquad \theta = \frac{\pi}{4} \qquad r = \frac{1}{\sin\theta}$$

$r\cos\theta = 1$

(d) The linear map which sends the unit square $[0, 1] \times [0, 1]$ to the parallelogram with vertices $(0, 0)$, $(6, 1)$, $(8, 5)$, and $(2, 4)$ is $G(u, v) =$

$$(6u + v, 2u + 4v) \qquad (6u + 2v, u + 4v) \qquad (6u + v, 4u + 2v)$$
$$(6u + 2v, 4u + v) \qquad (6u + 4v, u + 2v)$$



$(1,0) \to (6,1)$ $(0,1) \to (2,4)$, $(1,1) \to (8,5)$.

$$\begin{bmatrix} 6 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = 6u + 2v,\ u + 4v .$$