

Math 33B: Differential Equations

Midterm Exam 2

Wednesday, May 22, 2013 -9:00 - 9:50 AM

Instructor: Aliko M.

Name: _____

PLEASE PRINT

UID: _____

Section: _____

Discussion sections:

- *Tuesday* with: S. Kim - **1A** | F. Robinson - **1C** | J. Rooney - **1E**
- *Thursday* with: S. Kim - **1B** | F. Robinson - **1D** | J. Rooney - **1F**

Read the following information before starting the exam:

- Show **all** your work, clearly and in order;
- This test has **4 questions** and is worth a total of **50** points;
- No books, notes, electronic devices (inc. calculators) are allowed;
- Good luck!! ☺

QUESTION #	SCORE	MAX. POINTS
1		10
2		10
3		15
4		15
TOTAL		50

Question 1 (10 points)

(a) Suppose that $y_1 = x^{1/2}$ and $y_2 = x^{-1}$ are solutions to:

$$2x^2y'' + 3xy' - y = 0.$$

Show that the two solutions are linearly independent.

(b) Hence, find the general solution to the nonhomogeneous equation:

$$2x^2y'' + 3xy' - y = x^{-1}.$$

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Question 2 (10 points)

(a) Solve the initial value problem

$$y'' - y' - 2y = 0,$$

with $y(0) = \alpha$ and $y'(0) = 2$.

(b) Find the value of α such that the solution approaches zero as $t \rightarrow \infty$.

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Question 3 (15 points) Find the general solution to the following equation:

$$y'' + 5y' + 6y = e^{-3x}(1 + 4x).$$

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Question 4 (15 points)

Assume that the vertical spring-mass system described by the following harmonic motion equation:

$$my'' + \mu y' + ky = 0,$$

with $y(0) = u$ and $y'(0) = v$ is *critically* damped.

- (a) Find the general solution for $y(t)$ in terms of μ , m , u and v .
- (b) Show that if $v = 0$, $y \rightarrow 0$ as $t \rightarrow \infty$.
- (c) If $u > 0$, determine a condition on v in terms of μ , u and m that will ensure that the mass passes through its equilibrium point after it is released.
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