Math 33B: Differential Equations

Midterm Exam 1

Wednesday, Apr. 24, 2013 -9:00 - 9:50 AM *Instructor: Aliki M.*

Name: _____

PLEASE PRINT

UID: _____

Section: _____

Discussion sections:

- *Tuesday* with: S. Kim - **1A** | F. Robinson - **1C** | J. Rooney - **1E**
- *Thursday* with: S. Kim - **1B** | F. Robinson - **1D** | J. Rooney - **1F**

Read the following information before starting the exam:

- Show **all** your work, clearly and in order;
- This test has **5 questions** and is worth a total of **50** points;
- No books, notes, electronic devices (inc. calculators) are allowed;
- Good luck!! ☺

QUESTION #	SCORE	MAX. POINTS
1		10
2		10
3		10
4		10
5		10
TOTAL		**50**

Question 1 (*10 points*)

Using the substitution $y = vx$, where $v = v(x)$, find an explicit solution of the differential equation

$$(x^2 - y^2) + xy\frac{dy}{dx} = 0.$$

for which $y(1) = 1$.

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Question 2 (*10 points*)

Show that the differential equation,

$$(x - xy^2) + (6y - x^2y)\frac{dy}{dx} = 0$$

is exact.

Hence, find an explicit solution satisfying $y(2) = -2$.

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Question 3 (*10 points*)

Show that

$$\frac{d}{dx}\big(\ln[\sec x + \tan x]\big) = \sec x,$$

and hence, find the general solution of the *linear*, differential equation

$$\cos x \frac{dy}{dx} + y = 1 - \sin x$$

using an integrating factor.

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Question 4 (*10 points*)

A tank contains 40 gallons of fresh water. Brine with 3 lbs of salt per gallon flows in at the rate of 2 gal/min and the stirred mixture flows out at 3 gal/min.

(a) Formulate the IVP and solve it.

(b) Find the amount of salt in the tank when the liquid in it has been reduced to 20 gallons.

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Question 5 (*10 points*)

The following differential equation describes the growth of the population of the mouflon, an endangered species of sheep found on the island of Cyprus,

$$\frac{dy}{dt} = -ay(b-y)(c-y), \quad y(0) = y_0$$

where $y(t)$ denotes the population of the mouflon and a, b and c are positive constants and $b < c$.

(a) Determine the equilibrium points and classify each one as unstable, stable or semi-stable.

(b) Sketch the equilibrium and some non-equilibrium solutions on the $t - y$ plane.

(c) What should the initial value of the population, y_0, be for the species to avoid extinction?

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