MATH 33A

Midterm 1

Jan. 28, 2019

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DISCUSSION SECTION NUMBER: [redacted]

Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work—your work must both justify and clearly identify your final answer. No books, notes or calculators are allowed. You must simplify results of function evaluations when it is possible to do so.

For instructor use only

Page	Points	Score
2	6	6
3	5	5
4	13	12
5	8	6
6	8	6
7	10	10
Total:	50	35

1. [6 pts] Is the vector $\overrightarrow{\mathbf{b}} = \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$ a linear combination of the vectors $\overrightarrow{\mathbf{v}} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{matrix} 5 \\ 0 \\ 5 \end{matrix}$ and $\overrightarrow{\mathbf{w}} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{matrix} -9 \\ -3 \\ -3 \end{matrix}$? If so, write down the linear combination in the format $\overrightarrow{\mathbf{b}} = c_1 \overrightarrow{\mathbf{v}} + c_2 \overrightarrow{\mathbf{w}}$. If not, explain why not.

$\vec{b} = 5\vec{v} - 3\vec{w}$

yes

2. [5 pts] Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Circle all of the following vectors which are members of ker(A).

$$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \quad \vec{z} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$A(\vec{x}) = \vec{0}$

3. Suppose you know that $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$.

5 (a) [6 pts] Find $(AB)^{-1}$.

$(AB)^{-1} = B^{-1}A^{-1}$

$$\begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \overset{-1}{1} & 0 \\ 8 & 5 \end{pmatrix}$$

5 (b) [5 pts] Find B.

$$\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{-2-3}\begin{pmatrix} 2 & -1 \\ -3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2}{5} & \frac{1}{5} \\ 3/5 & \frac{1}{5} \end{pmatrix} ✓$$

$$\begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

2 (c) [2 pts] What was the rank of A? (this should require no computations)

2 ✓

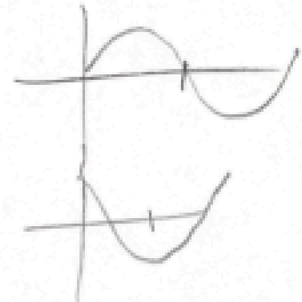
Proj, Refl, Rot

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that rotates all vectors about the origin *counter-clockwise* by $\pi/2$. Let $R : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects all vectors about the line $y = x$.

4 (a) [6 pts] Find the single matrix representing the composite function $T \circ R$. *Hint: A geometric approach may be easier than a computation.*

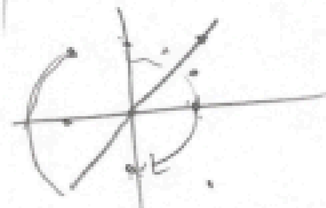
Rotates $\pi/2$ ccw → Reflects by $y=x$

$$\begin{bmatrix} \cos \pi/2 & \sin \pi/2 \\ \sin \pi/2 & \cos \pi/2 \end{bmatrix} \cdot \begin{vmatrix} 0 & +1 \\ +1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$



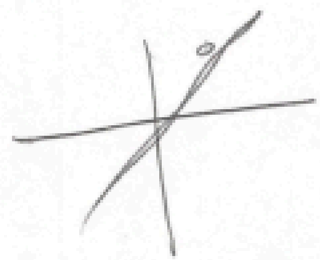
+2

flip x and y and sign



$$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \quad +2$$

$$\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \begin{vmatrix} -2 \\ 3 \end{vmatrix} = \begin{vmatrix} -3 \\ 2 \end{vmatrix}$$



$$\begin{vmatrix} w_1^2 & w_1 w_2 \\ w_1 w_2 & w_2^2 \end{vmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

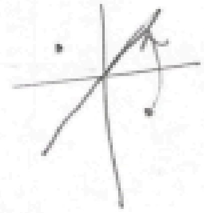
2 (b) [2 pts] Find $T(R(\vec{v}))$ where $\vec{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

$$\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \begin{vmatrix} -2 \\ 3 \end{vmatrix} = \begin{vmatrix} -2 \\ -3 \end{vmatrix}$$

(c) [6 pts] Find the single matrix representing the composite function $R \circ T$. *Hint: A geometric approach may be easier than a computation.*

Reflect about $y=x \rightarrow \pi/2$ ccw

$$\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$$

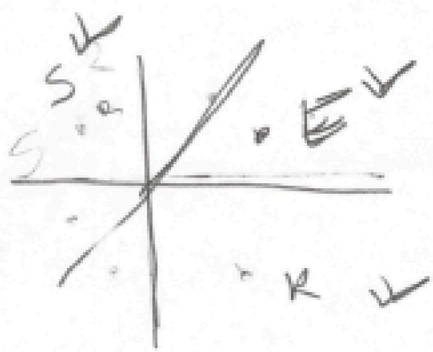


2

OK

(d) [2 pts] Find $R(T(\vec{\mathbf{v}}))$ where $\vec{\mathbf{v}} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

$$\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} -2 \\ 3 \end{vmatrix} = \begin{vmatrix} 2 \\ 3 \end{vmatrix}$$



5. [4 pts] Suppose you know that $\vec{w}$ is in $\ker(B)$, and you also know that $B\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Use this information to find $B(2\vec{w} - 3\vec{v})$.

$$B(2\vec{w} - 3\vec{v}) = B(2\vec{w}) - B(3\vec{v}) = 2\overset{0}{B(\vec{w})} - 3B(\vec{v})$$

$$= \begin{pmatrix} -3 \\ -6 \\ -9 \end{pmatrix}$$

6. True or false (circle your answer; no justification needed). In all of the problems below, A is an nxn square matrix.

(a) [2 pts] If A is the coefficient matrix for some linear system, and $\text{rank}(A) = n$, then the system has a unique solution.

TRUE FALSE

(b) [2 pts] If A is the coefficient matrix for some linear system, and $\text{rank}(A) < n$, then the system must have infinitely many solutions. no solution

TRUE FALSE

(c) [2 pts] If A is the coefficient matrix for some linear system, and the system had a unique solution, then the RREF of A must be precisely the identity matrix $I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$.

TRUE FALSE

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