6. [4 pts] Suppose you know that $\vec{w}$ is in ker(B), and you also know that $B\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Use this information to find $B(3\vec{w} - 2\vec{v})$.

$$B(3\vec{w}) - B(2\vec{v}) = 3B\vec{w} - 2B\vec{v}$$

since $\vec{w}$ in ker(B), $B\vec{w} = 0$

$$0 \qquad\qquad 0 - 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \boxed{\begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}}$$

4

7. True or false (circle your answer; no justification needed). In all of the problems below, A is ~~an~~ nxn *square* matrix.

(a) [2 pts] If A is the coefficient matrix for some linear system, and rank(A) $= n$, then the system has a unique solution.

6

TRUE FALSE

(b) [2 pts] If A is the coefficient matrix for some linear system, and rank(A) $< n$, then the system must have infinitely many solutions. or no solution

TRUE FALSE

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

(c) [2 pts] If A is the coefficient matrix for some linear system, and the system had a unique solution, then the RREF of A must be precisely the identity matrix $I_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}$.

TRUE FALSE

5. (a) [2 pts] Write down the 2x2 matrix for rotation by an angle θ.

$$A$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

7

(b) [2 pts] Use the determinant to show that this matrix is invertible.

$$\det(A) = \cos^2\theta + \sin^2\theta$$

$$\det(A) = 1$$

Since $\det(A) \neq 0$, and it is a square matrix, matrix is invertible

(c) [3 pts] Explain geometrically what the inverse matrix should do, and write the inverse matrix down.

Geometrically, the inverse should rotate it by an angle θ the opposite way (clockwise):

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{1} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

4. Suppose you know that $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$.

6

(a) [6 pts] Find $(AB)^{-1}$.

$(AB)^{-1} = B^{-1}A^{-1}$

2x2

$$\underset{2\times 2}{\begin{bmatrix} -1 & 1 \\ 3 & 2 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}} = \boxed{\begin{bmatrix} -1 & 0 \\ 8 & 5 \end{bmatrix}}$$

$$2\begin{bmatrix} -1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{matrix} -2+1 = -1 \\ 6+2 = 8 \end{matrix}$$

$$1\begin{bmatrix} -1 \\ 3 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{matrix} -1+1 = 0 \\ 3+2 = 5 \end{matrix}$$

6

(b) [6 pts] Find B.

$B = (B^{-1})^{-1}$

$$(B^{-1})^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$B = \frac{1}{(-1)(2)-(3)(1)}\begin{bmatrix} 2 & -1 \\ -3 & -1 \end{bmatrix}$$

$$B = -\frac{1}{5}\begin{bmatrix} 2 & -1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -2/5 & 1/5 \\ 3/5 & 1/5 \end{bmatrix}$$

2

(c) [2 pts] What was the rank of A? (this should require no computations)

2

5

3. [5 pts] Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

$\ker(A) : A\vec{x} = 0$

if $\vec{x} = 0$, then its part of $\ker(A)$

Circle all of the following vectors which are members of ker(A).

$$\vec{\mathbf{v}} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \vec{\mathbf{w}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{\mathbf{x}} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \quad \vec{\mathbf{y}} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \quad \vec{\mathbf{z}} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{\substack{+(I) \\ -3(I) \\ -(I)}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{bmatrix} \xrightarrow{\div 2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{bmatrix} \xrightarrow{\substack{-2(II) \\ 2(II) \\ (II)}} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -3 \end{bmatrix} \rightarrow \div -2$$

$$\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{\substack{(I) \\ -3(I) \\ -(I)}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{bmatrix} \xrightarrow{\substack{-2(II) \\ 2(II) \\ -2(II)}} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_3 = 0$

$x_2 = 0$

$x_4 = 0$

Let $x_3 = s$

$x_1 = s$

$x_2 = 0$

$x_3 = s$

$x_4 = 0$

span $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{\substack{(I) \\ -3(I) \\ -(I)}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\substack{-2(II) \\ -(II) \\ -(II)}} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_3 - x_4 = 0$

$x_2 + x_4 = 0$

$x_1 = x_3 + x_4$

$x_2 = -x_4$

Let $x_3 = s$

$x_4 = t$

$x_1 = s + t$

$x_2 = -t$

$x_3 = s$

$x_4 = t$

$\ker(A) = \text{span} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

6

2. [6 pts] Is the vector $\vec{b} = \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$ a linear combination of the vectors $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$? If so, write down the linear combination in the format $\vec{b} = c_1\vec{v} + c_2\vec{w}$. If not, explain why not.

$$\vec{b} = c_1\vec{v} + c_2\vec{w}$$

$$b = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 1 & -3 \\ 1 & 1 & 2 \end{array}\right] \xrightarrow{-(\text{I})} \left[\begin{array}{cc|c} 1 & 3 & -4 \\ 0 & 1 & -3 \\ 0 & -2 & 6 \end{array}\right] \xrightarrow[2(\text{II})]{-3(\text{II})} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

$c_1 = 5$
$c_2 = -3$ $\longrightarrow$ $\vec{b} = 5\vec{v} - 3\vec{w}$

$$\begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 - 9 \\ 0 - 3 \\ 5 - 3 \end{bmatrix} = \begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} \checkmark$$

Yes, $\vec{b}$ is a linear combo of $\vec{v}$ and $\vec{w}$

1. Consider the matrix

$$A = \begin{pmatrix} 1 & 4 & -2 \\ 1 & 4 & -2 \end{pmatrix}$$

Recall that A corresponds to a linear transformation T_A.

(a) [2 pts] What are the domain and range of T_A? Domain $= \ker(A)$ Range $= \mathrm{Im}(A)$

$\left[\begin{array}{ccc|c} 1 & 4 & -2 & \\ 1 & 4 & -2 & \end{array}\right] \xrightarrow{(-1)} \begin{bmatrix} 1 & 4 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 + 4x_2 - 2x_3 = 0$

$x_1 = -4x_2 + 2x_3$

Let $x_2 = s$
$x_3 = t$

$x_1 = -4s + 2t$
$x_2 = s$
$x_3 = t$

Domain $=$ span $\left(\begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}\right)$

-1

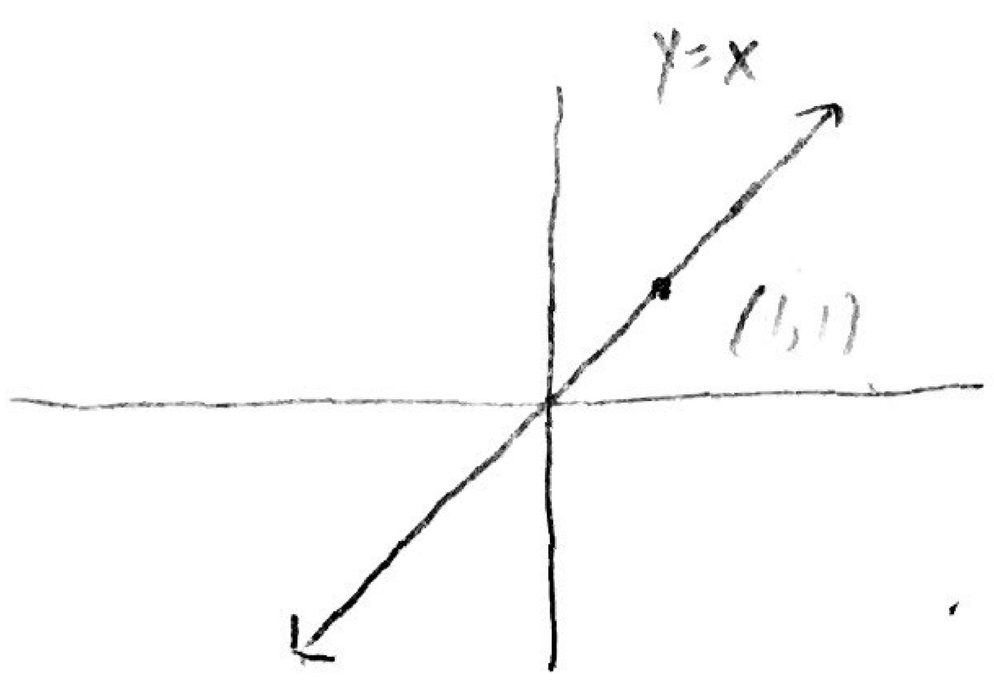
Range $=$ span $\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$ OK

(b) [2 pts] Describe the image of T_A as a span of vector(s).

Image of T_A = span $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(c) [4 pts] Describe the image of T_A geometrically. Is it a line? A plane? Draw it.

The image of T_A is a line because it is a span of only one vector. One vector cannot make a plane, but two can



MATH 33A | Midterm 1 | Oct. 23, 2017

STUDENT NAME: Christian Rodriguez

STUDENT ID NUMBER: 804789345

DISCUSSION SECTION NUMBER: Disc 2A

Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work—your work must both justify and clearly identify your final answer. No books, notes or calculators are allowed. You must simplify results of function evaluations when it is possible to do so.

For instructor use only

Page	Points	Score
2	8	7
3	6	6
4	5	5
5	14	14
6	7	7
7	10	10
Total:	50	49