MATH 33A Final Exam Dec. 11, 2017

STUDENT NAME: Answer Key

STUDENT ID NUMBER: ____________________

DISCUSSION SECTION NUMBER: ____________________

Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work—your work must both justify and clearly identify your final answer. No books, notes or calculators are allowed. You must simplify results of function evaluations when it is possible to do so.

For instructor use only

Page	Points	Score
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1. Suppose A is a 4x6 matrix, and B is a 6x4 matrix.

   (a) [2 pts] What is the size (dimension x dimension) of the matrix BA?

   6×6

   (b) [2 pts] What are the possible values for the rank of BA?

   $$\begin{tikzcd} \mathbb{R}^6 \arrow[r, "A"] \arrow[rr, bend right, "BA"'] & \mathbb{R}^4 \arrow[r, "B"] & \mathbb{R}^6 \end{tikzcd}$$

   $\text{RANK}(BA) = 4, 3, 2, 1, 0$

   (c) [2 pts] What are the possible values for the nullity of BA?

   $\text{NULLITY}(BA) = 2, 3, 4, 5, 6$

   (d) [2 pts] Can BA be invertible? Explain.

   NO. $\text{RANK}(BA) \neq 6$.

2. Suppose $\vec{v}$ and $\vec{w}$ are two *unit* vectors satisfying $\vec{v} \cdot \vec{w} = \frac{1}{2}$.

(a) [2 pts] Find the angle between $\vec{v}$ and $\vec{w}$.

$\vec{V} \cdot \vec{\omega} = \frac{1}{2} = \|\vec{v}\| \|\vec{\omega}\| \cos\theta \qquad \|\vec{v}\| = 1 = \|\vec{\omega}\|$

$\frac{1}{2} = \cos\theta \qquad \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

(b) [3 pts] If M is an *orthogonal* matrix, find the dot product $(M(\vec{v} - \vec{w})) \cdot (M\vec{w})$.

$M(\vec{v} - \vec{\omega}) \cdot M\vec{\omega} = (\vec{V} - \vec{\omega}) \cdot \vec{\omega}$ (M IS ORTHOGONAL)

$= \vec{V} \cdot \vec{\omega} - \vec{\omega} \cdot \vec{\omega}$

$= \frac{1}{2} - 1 = -\frac{1}{2}$

3. [3 pts] My linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ took the unit circle and stretched it into an ellipse having an area of 3. If A is the matrix of my linear transformation, what is $\det(A)$?

$\text{AREA}(T(\Omega)) = |\text{DET } A| \text{ AREA}(\Omega)$. LET Ω = UNIT CIRCLE WITH AREA $= \pi$

$3 = |\text{DET } A| \, \pi \Longrightarrow \text{DET } A = \pm \frac{3}{\pi}$

4. [4 pts] If I perform a QR-Factorization on an $n \times n$ matrix B, what are $\det(Q)$ and $\det(R)$ in terms of $\det(B)$?

$B = QR$

$\text{DET}(B) = \text{DET}(QR) = \text{DET}(Q)\,\text{DET}(R)$

Q ORTHOGONAL $\Longrightarrow \text{DET}(Q) = \pm 1$

$\longrightarrow \text{DET}(R) = \pm \text{DET}(B)$

5. Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 2 \end{pmatrix}$$

(a) [4 pts] Find a basis for $\mathrm{Im}(A)$.

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 3 & 2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 3 & 2 \end{pmatrix} \begin{matrix} \\ -2R1 \\ -3R1 \end{matrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & -3 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

WILL GET PIVOT IN FIRST + SECOND COLUMN.

$$\Longrightarrow \left\{ \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\} \text{ IS BASIS FOR } \mathrm{Im}(A)$$

(b) [4 pts] Find a basis for $\ker(A)$.

CONTINUING

$$\begin{array}{c} \\ \\ \end{array} \longrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{-2R2} \begin{array}{c} \begin{array}{ccc} & & s \\ & & \downarrow \end{array} \\ \begin{pmatrix} 1 & 0 & 1/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

$\vec{x}_1 = -1/3\, s$
$\vec{x}_2 = -1/3\, s$
$\vec{x}_3 = s$

$$\Longrightarrow \left\{ \begin{pmatrix} -1/3 \\ -1/3 \\ 1 \end{pmatrix} \right\} \text{ IS BASIS FOR } \ker(A)$$

(c) [2 pts] Use your previous work to write a linear relation between the columns of A.

$$-\frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + -\frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

6. [10 pts] Find the QR-Factorization for the matrix below:

$$A = \begin{pmatrix} 3 & 9 & 1 \\ 3 & -1 & 2 \\ 3 & -1 & 0 \\ 3 & 9 & 1 \end{pmatrix}$$

$\|\vec{v}_1\| = \sqrt{36} = 6$

$\vec{u}_1 = \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$

$\vec{u}_1 \cdot \vec{v}_2 = \frac{1}{2}(9-1-1+9) = 8$

$\vec{v}_2^{\perp} = \vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1 = \begin{pmatrix} 9 \\ -1 \\ -1 \\ 9 \end{pmatrix} - 8 \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ -1 \\ -1 \\ 9 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ -5 \\ 5 \end{pmatrix}$

$\|\vec{v}_2^{\perp}\| = \sqrt{100} = 10$

$\vec{u}_2 = \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$

$\vec{u}_1 \cdot \vec{v}_3 = \frac{1}{2}(1+2+0+1) = 2$

$\vec{u}_2 \cdot \vec{v}_3 = \frac{1}{2}(1-2+0+1) = 0$

$\vec{v}_3^{\perp} = \vec{v}_3 - (\vec{u}_1 \cdot \vec{v}_3)\vec{u}_1 - (\vec{u}_2 \cdot \vec{v}_3)\vec{u}_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - 0\vec{u}_2$

$= \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$

$\|\vec{v}_3^{\perp}\| = \sqrt{2}$

$\vec{u}_3 = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix} \longrightarrow A = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 0 \end{pmatrix}\begin{pmatrix} 6 & 8 & 2 \\ 0 & 10 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$

7. [4 pts] Find the determinant of the following matrix:

$$A = \begin{pmatrix} 2 & 4 & 4 & 2 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{\text{SWAP}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 4 & 2 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{-2R1}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{-\frac{1}{2}R2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 1 & 2 \end{pmatrix} \xrightarrow{-\frac{1}{2}R3}$$

$$\frac{4}{2} - \frac{9}{2} = \frac{-5}{2}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & -\frac{5}{2} \end{pmatrix}$$

$$\mathrm{DET}(A) = -1^5 \cdot \mathrm{DET}(\mathrm{THIS}) = -1 \cdot -10 = 10$$

8. (a) [4 pts] For an $n \times n$ matrix A, and a scalar k, explain why $\det(kA) = k^n \det(A)$.

kA MULTIPLIES ALL n ROWS BY k. SINCE MULTIPLYING ANY SINGLE ROW AFFECTS THE DETERMINANT BY MULTIPLICATION, WE GET

$$\text{DET}(kA) = k^n \, \text{DET}(A)$$

(b) [6 pts] Show that it is impossible for a 3×3 matrix to be both skew symmetric and orthogonal. *Hint: You may make use of the previous part of the problem, even if you couldn't show it.*

SKEW SYM $\rightsquigarrow A^T = -A \rightsquigarrow \text{DET}(A^T) = \text{DET}(-A)$

ORTHOGONAL $\rightsquigarrow A^T = A^{-1} \rightsquigarrow A^T A = I$

$\rightsquigarrow \text{DET}\, A = \pm 1$

$\text{DET}(A) = \text{DET}(-A)$

$\text{DET}(A) = (-1)^3 \, \text{DET}(A)$

$\text{DET}(A) = 0$

CONTRADICTION

9. Let A be a 3x3 skew symmetric matrix.

(a) [4 pts] Explain why the trace of A must be 0.

IF $A^T = -A$, THE DIAGONAL ELEMENTS OF A DON'T CHANGE WHEN TAKING TRANSPOSE, INDICATING $d = -d$ FOR ANY d ON DIAGONAL OF A

$\Longleftrightarrow d = 0$ FOR ANY d ON DIAGONAL OF A

$\Longrightarrow Tr(A) = 0$

(b) [6 pts] If one of the complex eigenvalues of A is $\lambda_1 = 1+2i$, find all of the other eigenvalues.

$\lambda_1 = 1+2i$ FORCES $\lambda_2 = 1-2i$

THEN $Tr A = 0$ FORCES $1+2i + 1-2i + \lambda_3 = 0$

$\Longrightarrow \lambda_3 = -2$

(c) [3 pts] Find $\det(A)$.

$DET(A) = \lambda_1 \lambda_2 \lambda_3 = (1+2i)(1-2i)(-2)$

$= (1+4)(-2)$

$= -10$

10. (a) [4 pts] Give the definition for a collection of n vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ to be linearly independent.

IF $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n = \vec{0}$, THEN ALL c's $= 0$

(b) [8 pts] If a collection of n non-zero vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are mutually orthogonal ($\vec{v}_i \perp \vec{v}_j$ as long as $i \neq j$), prove that they are also linearly independent.

SUPPOSE $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n = \vec{0}$. DOT BOTH SIDES BY $\vec{v}_1$.

$$c_1\vec{v}_1\cdot\vec{v}_1 + c_2\underbrace{\cancel{\vec{v}_1\cdot\vec{v}_2}}_{0} + \cdots + c_n\underbrace{\cancel{\vec{v}_1\cdot\vec{v}_n}}_{0} = \underbrace{\cancel{\vec{v}_1\cdot\vec{0}}}_{0}$$

$$c_1\vec{v}_1\cdot\vec{v}_1 = 0$$

THEN $\vec{v}_1 \neq \vec{0}$ MEANS $\vec{v}_1\cdot\vec{v}_1 \neq 0$, FORCING $c_1 = 0$.

SIMILARLY FOR $\cdot\vec{v}_2 \rightsquigarrow c_2 = 0$

$\cdot\vec{v}_3 \rightsquigarrow c_3 = 0$

$\vdots$

$\cdot\vec{v}_n \rightsquigarrow c_n = 0$

(c) [4 pts] Is it possible to find 5 mutually orthogonal non-zero vectors in $\mathbb{R}^4$? If yes, give an example. If not, explain why not. You may use the previous part of the problem even if you were unable to prove it.

NO. MUTUAL ORTHOG $\Longrightarrow$ LINEARLY INDEP. (PART b)

BUT WE CANNOT HAVE 5 LIN. INDEP. VECTORS IN $\mathbb{R}^4$

(THEY WOULD SPAN A 5-DIM SUBSPACE OF $\mathbb{R}^4$, IMPOSSIBLE)

11. [10 pts] For the matrix A below, find eigenvalues, then corresponding eigenspaces, and then diagonalize.

$$A = \begin{pmatrix} 0 & -3 \\ 2 & 5 \end{pmatrix}$$

$$f_A(\lambda) = \lambda^2 - 5\lambda + 6 = (\lambda-3)(\lambda-2) \qquad \lambda = 2,\ \lambda = 3$$

$$E_2 = \ker \begin{pmatrix} -2 & -3 \\ 2 & 3 \end{pmatrix} \text{ WITH BASIS } \left\{ \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right\}$$

$$\begin{array}{ccc} \mathcal{E} & \xrightarrow{A} & \mathcal{E} \\ \uparrow S & & \uparrow S \\ \mathcal{B} & \xrightarrow{B} & \mathcal{B} \end{array}$$

$$E_3 = \ker \begin{pmatrix} -3 & -3 \\ 2 & 2 \end{pmatrix} \text{ WITH BASIS } \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$

$$A = \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix}^{-1}$$

12. [8 pts] Let b and c be two positive numbers. Suppose we have a 2x2 matrix of the form

$$A = \begin{pmatrix} 0 & b^2 \\ -c^2 & 0 \end{pmatrix}$$

Show that A is similar to a rotation-scaling matrix; find the rotation-scaling matrix in terms of b and c. What is the scaling factor of this matrix, and what is the angle of rotation?

$$f_A(\lambda) = \lambda^2 + b^2c^2 = (\lambda + bci)(\lambda - bci) \quad \left(\text{or } \lambda = \frac{0 \pm \sqrt{0 - 4b^2c^2}}{2} = \pm bci \right)$$

USE $\lambda = bci$, FIND COMPLEX EIGENVECTOR:

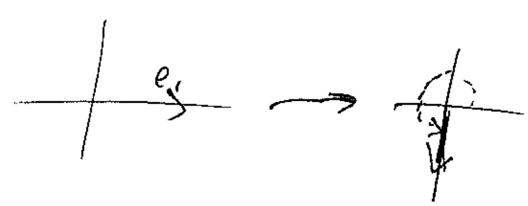
$$\text{KER} \begin{pmatrix} -bci & b^2 \\ -c^2 & -bci \end{pmatrix} \text{ HAS BASIS } \left\{ \begin{pmatrix} b^2 \\ bci \end{pmatrix} = \underline{\underline{\overset{\vec{v}}{\begin{pmatrix} b^2 \\ 0 \end{pmatrix}} + i \overset{\vec{w}}{\begin{pmatrix} 0 \\ bc \end{pmatrix}}}} \right\}$$

$$\text{SO } A = \begin{pmatrix} b^2 & 0 \\ 0 & bc \end{pmatrix} \begin{pmatrix} 0 & bc \\ -bc & 0 \end{pmatrix} \begin{pmatrix} b^2 & 0 \\ 0 & bc \end{pmatrix}^{-1}$$

SCALING FACTOR $= bc$

ROTATION ANGLE $= \frac{3\pi}{2}$ $\left(\tan^{-1}\left(\frac{bc}{0}\right) = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \ldots \quad \vec{e}_1 \rightsquigarrow \begin{pmatrix} 0 \\ -bc \end{pmatrix} \right)$

NEGATIVE!!



13. Let V be a subspace of $\mathbb{R}^n$, with $\dim(V) = m$. Let P_V be the matrix of orthogonal projection onto V.

(a) [6 pts] List the eigenvalues of P_V, and for each one, give the algebraic multiplicity, the geometric multiplicity, and the corresponding eigenspace.

$$\left.\begin{array}{l} \lambda = 1 \quad E_1 = V, \text{ so } GeMu(1) = m \\ \lambda = 0 \quad E_0 = V^{\perp}, \text{ so } GeMu(0) = n-m \end{array}\right\} \text{FORCES} \quad \begin{array}{l} AlMu(1) = m \\ AlMu(0) = n-m \end{array}$$

(b) [4 pts] State the Spectral Theorem.

A MATRIX IS ORTHOGONALLY DIAGONALIZABLE IF & ONLY IF IT IS SYMMETRIC

(c) [5 pts] Explain why the Spectral Theorem guarantees that P_V must be a symmetric matrix (do not use a formula for P_V).

THE EIGENSPACES E_1 AND E_0 ARE ORTHOGONAL, SO P_V MUST BE ORTHOGONALLY DIAGONALIZABLE, THUS P_V IS SYMMETRIC

(d) [4 pts] Is the quadratic form $q(\vec{x}) = \vec{x} \cdot P_V \vec{x}$ positive definite, positive semi-definite, or indefinite?

POSITIVE SEMI-DEFINITE

14. (a) [2 pts] Write the quadratic form $q(x) = 6x_1^2 + 4x_1x_2 + 3x_2^2$ in matrix format.

$$q(\vec{x}) = \vec{x} \cdot \underbrace{\begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix}}_{A} \vec{x}$$

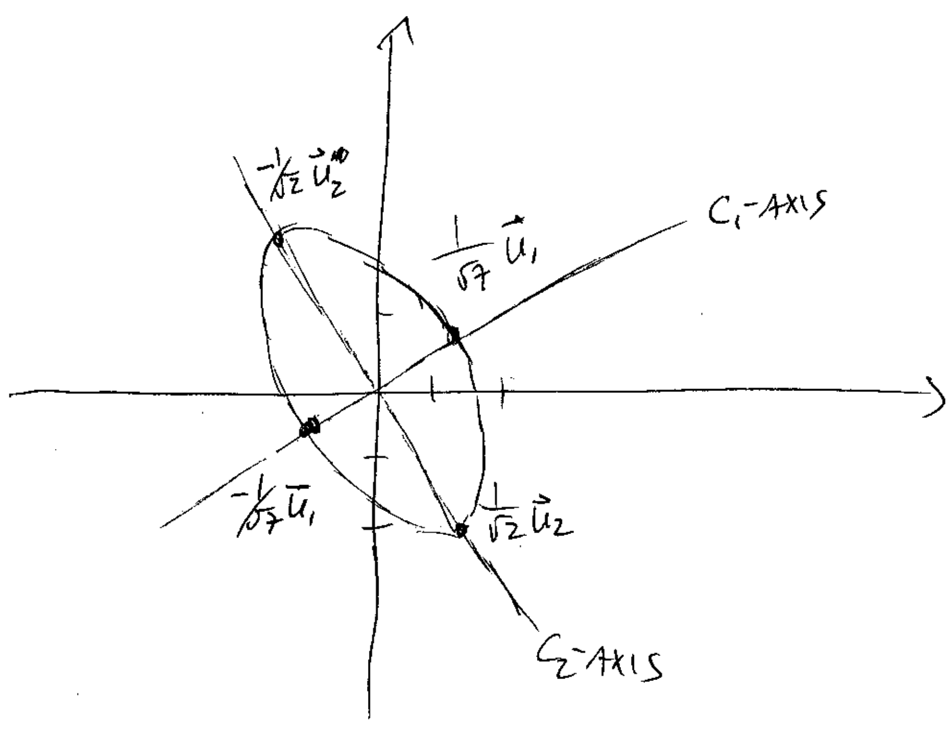
(b) [8 pts] Sketch the curve $q(x) = 1$, labelling the principal axes and the intercepts of the curve with the principal axes.

$$f_A(\lambda) = \lambda^2 - 9\lambda + 14 = (\lambda - 7)(\lambda - 2)$$

$$E_7 = \ker \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix} \text{ WITH BASIS } \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\} \rightsquigarrow \vec{u}_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$E_2 = \ker \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \text{ WITH BASIS } \left\{ \begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\} \to \vec{u}_2 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$q(\vec{x}) = 7c_1^2 + 2c_2^2 = 1$$



15. (a) [6 pts] Prove that, for any $n \times n$ matrix A, the characteristic polynomials $f_A(\lambda)$ and $f_{A^T}(\lambda)$ are the same, and thus A and A^T have the same eigenvalues. *Hints: $I = I^T$, algebraic properties of determinant, algebraic properties of transpose*

$$f_A(\lambda) = \text{DET}(A - \lambda I)$$
$$= \text{DET}\left((A-\lambda I)^T\right)$$
$$= \text{DET}\left(A^T - \lambda I^T\right)$$
$$= \text{DET}\left(A^T - \lambda I\right) = f_{A^T}(\lambda)$$

(b) [6 pts] If every row of A is made up of entries that add up to 1 (the matrix $\begin{pmatrix} 0 & 1 \\ .2 & .8 \end{pmatrix}$ is an example), prove that $\lambda = 1$ is an eigenvalue of A. *Hint: Find a clever eigenvector that will allow this to happen regardless of A; starting from the example might provide some ideas*

USING $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$, $A\vec{v} = \begin{pmatrix} & & \\ & A & \\ & & \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$

$$= \begin{pmatrix} \text{SUM OF ROW 1} \\ \text{SUM OF ROW 2} \\ \vdots \\ \text{SUM OF ROW } n \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = 1\vec{v}$$

SO $\lambda = 1$ IS EIGENVALUE

(c) [2 pts] Use the previous parts of the problem to conclude that, for a matrix with *columns* having entries that add up to 1, the value $\lambda = 1$ will always be an eigenvalue.

IF A HAS COLUMNS ADDING TO 1, THEN A^T HAS ROWS ADDING TO 1, AND THUS A^T HAS $\lambda=1$ AS AN EIGENVALUE (PART b), THUS A HAS $\lambda=1$ AS AN EIGENVALUE (PART a)

16. Consider the following matrix:

$$A = \begin{pmatrix} 1 & 12 & \frac{2}{3} & 0 & \cos(1) \\ 3 & 2 & -7 & 0 & \frac{4}{\pi} \\ -5 & 6 & 0 & 0 & 2 \\ 3 & e^{-e} & \ln 5 & 0 & -1 \\ 1 & \frac{\pi}{\sqrt{2}} & 1 & 0 & 1 \end{pmatrix}$$

(a) [2 pts] How many eigenvalues (real and complex) does A have, counting multiplicities?

5

(b) [2 pts] What is the sum of those eigenvalues?

SUM = TR(A) = 4

(c) [3 pts] What is the product of those eigenvalues? Explain.

PRODUCT = DET$(A) = 0$ BECAUSE THE COLUMN OF ZERO'S FORCES A NOT INVERTIBLE

17. [3 pts] Let A be a diagonal $n \times n$ matrix with $\text{rank}(A) = r$, where $r < n$. Find the geometric multiplicity of the eigenvalue $\lambda = 0$ for A in terms of n and r.

$$\text{GEMU}(0) = \text{DIM}(\text{KER}(A - 0I))$$
$$= \text{DIM}(\text{KER } A)$$
$$= n - \text{DIM}(\text{IM } A) \quad (\text{RANK-NULLITY})$$
$$= n - r$$

18. True or false (circle your answer; no justification needed).

(a) [2 pts] If the matrix A is similar to B, then A^3 is similar to B^3.

TRUE FALSE

$A = SBS^{-1} \rightsquigarrow A^3 = SBS^{-1}SBS^{-1}SBS^{-1}$
$= SB^3S^{-1}$

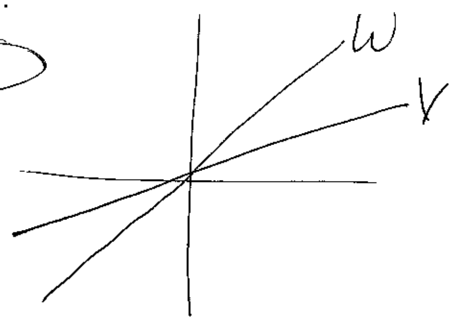
(b) [2 pts] If an $n \times n$ matrix A is *not* invertible, then $\ker(A)$ is an eigenspace of A.

TRUE FALSE

NOT INVERTIBLE $\rightsquigarrow$ 0 IS AN EIGENVALUE
+ $\text{KER}(A - 0I)$ IS EIGENSPACE

(c) [2 pts] If two subspaces of $\mathbb{R}^n$ intersect at the origin only, then the subspaces are orthogonal complements of each other.

TRUE FALSE



$V \cap W$ = ORIGIN

BUT V NOT $\perp$ W