33A Visan | Midterm 1 | February 8th

First Name: ______________________ ID#______________________

Last Name: ______________________

Section: ______________________ $= \begin{cases} 1a & \text{Tuesday with M. Bulkow} \\ 1b & \text{Thursday with M. Bulkow} \\ 1c & \text{Tuesday with L. Vera} \\ 1d & \text{Thursday with L. Vera} \\ 1e & \text{Tuesday with A. Mennen} \\ 1f & \text{Thursday with A. Mennen} \end{cases}$

Rules.

- There are **FOUR** problems, **each worth 10 points.**
- No calculators, computers, notes, books, e.t.c....
- Use the backs of the pages. There are two spare pages at the back.
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, or snoring. Try to sit still.
- Turn off your cell-phone.

1	2	3	4	Σ

(1) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 0 \\ 3 & 1 & 5 \end{bmatrix}.$$

(a) Find the general solution to the linear system

$$A\vec{x} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}.$$

(b) What is the rank of A?

(c) Indicate a basis for the image of A.

(d) There exists $\vec{b} \in \mathbb{R}^3$ for which the system $A\vec{x} = \vec{b}$ has no solution. TRUE/FALSE

(e) What is the dimension of the kernel of A?

(2) (a) Find a nonzero 3×3 matrix A such that $A\vec{x}$ is perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for all $\vec{x} \in \mathbb{R}^3$.

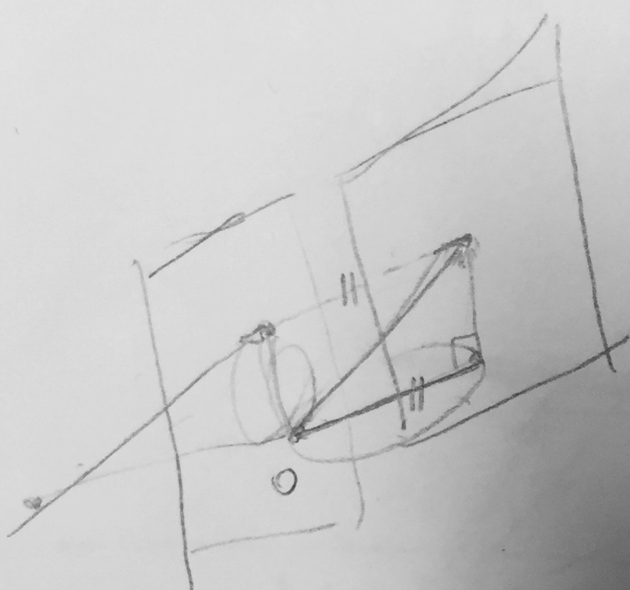
(b) Indicate a basis for the kernel of A.

(c) Indicate a basis for the image of A.

(3) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right)=\begin{bmatrix}1\\1\\0\end{bmatrix},\quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}3\\0\\4\end{bmatrix},\quad \text{and}\quad T\left(\begin{bmatrix}-1\\1\\0\end{bmatrix}\right)=\begin{bmatrix}-2\\4\\2\end{bmatrix}.$$

What is the matrix representing T?



$\vec{x} =$

(4) Consider two bases for $\mathbb{R}^2$, namely, $\mathcal{B} = (\vec{v}_1, \vec{v}_2)$ and $\mathcal{G} = (\vec{w}_1, \vec{w}_2)$, where

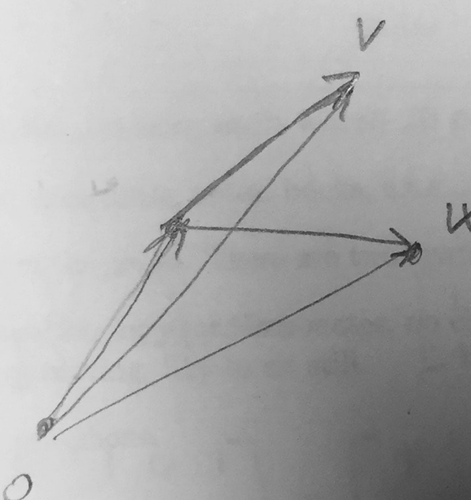
$$\vec{v}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \text{and} \quad \vec{w}_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \quad \vec{w}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}.$$

(a) Find $[\vec{v}_1]_{\mathcal{G}}$ and $[\vec{v}_2]_{\mathcal{G}}$.

(b) If the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies $[T(\vec{x})]_{\mathcal{G}} = [\vec{x}]_{\mathcal{B}}$ for all $\vec{x} \in \mathbb{R}^2$, find $[T]_{\mathcal{G}}$.

$3r^2 + 5s^2 - 44s + 4rs + 242$

$4+16+4+9 = 33$



$$\left\{ \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \right\}$$

$$\left\{ \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} ;\ t, s \in \mathbb{R} \right\}$$

$$\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 5 \end{bmatrix}$$