1C

Mathematics 33A Exam #1 Fall Quarter 2016

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Instructions:

- You have 50 minutes for this exam.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers. If you use the back of a page please indicate that you have work on the reverse side.
- You may not use books, notes, calculators, mobile phones, or any outside help during this exam. You may not collaborate with other students in any way during this exam.

Problem	Points	Score
1	10	4
2	10	10
3	10	10
4	10	8
Total	40	32

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1. (10 points total) Circle "T" if the statement is true and "F" if false. You need not show your work. No partial credit will be given.

   a. (2 points) If A and B are matrices such that $C = AB$ exists and C^{-1} exists, then A and B are both invertible.

   T F

   $CC^{-1} = I_n$ $\qquad$ $C^{-1}C = I_n$

   $(AB)C^{-1} = I_n$ $\qquad$ $C^{-1}(AB) = I_n$

   $A(BC^{-1}) = I_n$ $\qquad$ $(C^{-1}A)B = I_n$

   b. (2 points) If $AB = 0$ then it is always true that either $A = 0$ or $B = 0$. Here 0 is the zero matrix.

   T F

   c. (2 points) If A is an $n \times n$ matrix such that $A^2 = A$, and A is not the identity matrix I_n, then $\ker(I_n - A) = \mathrm{im}(A)$.

   T F

   d. (2 points) There is a square matrix whose image equals its kernel.

   T F $\qquad$ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

   e. (2 points) $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ x \\ y-1 \end{pmatrix}$ is a linear transformation.

   T F

10

2. (10 points total) The following problem has parts a) and b):

a. (5 points) Find the reduced row-echelon form of

$$\begin{pmatrix}1&6&7&1&0&0\\1&6&6&0&1&0\\1&5&4&0&0&1\end{pmatrix}\begin{matrix}\\-r_1\\-r_1\end{matrix}$$

$$\begin{bmatrix}1&6&7&1&0&0\\0&0&-1&-1&1&0\\0&-1&-3&-1&0&1\end{bmatrix}\begin{matrix}+6r_3\\\\\\\end{matrix}$$

$$\begin{bmatrix}1&0&-11&-5&0&6\\0&0&-1&-1&1&0\\0&-1&-3&-1&0&1\end{bmatrix}$$

$$\begin{bmatrix}1&0&-11&-5&0&6\\0&-1&-3&-1&0&1\\0&0&-1&-1&1&0\end{bmatrix}\begin{matrix}\\\div -1\\\div -1\end{matrix}$$

$$\begin{bmatrix}1&0&-11&-5&0&6\\0&1&3&1&0&-1\\0&0&1&1&-1&0\end{bmatrix}\begin{matrix}+11r_3\\-3r_3\\\\\end{matrix}$$

$$\boxed{\begin{bmatrix}1&0&0&6&-11&6\\0&1&0&-2&3&-1\\0&0&1&1&-1&0\end{bmatrix}}$$

✓

b. (5 points) Solve the equation $A\vec{x} = \vec{b}$ for $\vec{x}$ when

$$A = \begin{pmatrix}6&-11&6\\-2&3&-1\\1&-1&0\end{pmatrix} \text{ and } \vec{b} = \begin{pmatrix}1\\1\\1\end{pmatrix}.$$

$$\left[\begin{array}{ccc|c}6&-11&6&1\\-2&3&-1&1\\1&-1&0&1\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&-1&0&1\\-2&3&-1&1\\6&-11&6&1\end{array}\right]\begin{matrix}\\+2r_1\\-6r_1\end{matrix}$$

$$\left[\begin{array}{ccc|c}1&-1&0&1\\0&1&-1&3\\0&-5&6&-5\end{array}\right]\begin{matrix}+r_2\\\\+5r_2\end{matrix}$$

$$\left[\begin{array}{ccc|c}1&0&-1&4\\0&1&-1&3\\0&0&1&10\end{array}\right]\begin{matrix}+r_3\\+r_3\\\\\end{matrix}$$

$$\left[\begin{array}{ccc|c}1&0&0&14\\0&1&0&13\\0&0&1&10\end{array}\right]$$

$$\boxed{\vec{x} = \begin{bmatrix}14\\13\\10\end{bmatrix}}$$

✓

3. (10 points total) Let $V = span(v_1, v_2, v_3, v_4, v_5)$, where

$$v_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}, v_2 = \begin{pmatrix}2\\3\\4\end{pmatrix}, v_3 = \begin{pmatrix}1\\1\\1\end{pmatrix}, v_4 = \begin{pmatrix}3\\7\\11\end{pmatrix}, v_5 = \begin{pmatrix}3\\4\\5\end{pmatrix}.$$

a. (2 points) Are the vectors whose span defines V linearly independent or linearly dependent? Explain why or why not.

These vectors are linearly dependent because there are redundant vectors such as $\vec{v_2} = \vec{v_1} + \vec{v_3}$.

b. (4 points) Find a basis for V.

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 3 \\ 2 & 3 & 1 & 7 & 4 \\ 3 & 4 & 1 & 11 & 5 \end{bmatrix} \begin{matrix} \\ -2r_1 \\ -3r_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 3 \\ 0 & -1 & -1 & 1 & -2 \\ 0 & -2 & -2 & 2 & -4 \end{bmatrix} \begin{matrix} \\ \div -1 \\ \\ \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 3 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & -2 & -2 & 2 & -4 \end{bmatrix} \begin{matrix} -2r_2 \\ \\ +2r_2 \end{matrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -1 & 5 & -1 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

omit $\vec{v_3}, \vec{v_4}, \vec{v_5}$

$\boxed{\text{basis for } V \text{ is the span}(\vec{v_1}, \vec{v_2})}$

c. (4 points) Find the kernel of the matrix $A = [v_1\ v_2\ v_3\ v_4\ v_5]$.

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 3 & 0 \\ 2 & 3 & 1 & 7 & 4 & 0 \\ 3 & 4 & 1 & 11 & 5 & 0 \end{array}\right]$$

$\Downarrow$ from part (b)

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1 & 5 & -1 & 0 \\ 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = x_3 - 5x_4 + x_5$

$x_2 = -x_3 + x_4 - 2x_5$

let $x_3 = t$

$x_4 = \Delta$

$x_5 = u$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} t - 5\Delta + u \\ -t + \Delta - 2u \\ t \\ \Delta \\ u \end{bmatrix} = t\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \Delta\begin{bmatrix} -5 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\ker(A) = \text{span}\left(\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right)}$$

4. (10 points total) The following problem has parts a) and b):

a. (6 points) Find the 3×3 matrix which represents orthogonal projection onto the line spanned by $v = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$.

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$\text{proj}_L(\vec{x}) = (\vec{x} \cdot \vec{u})\vec{u}$

$\vec{u} = \frac{1}{\sqrt{1^2+3^2+4^2}} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$\vec{u} = \frac{1}{\sqrt{26}} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$\text{proj}_L(\vec{x}) = \left(\vec{x} \cdot \frac{1}{\sqrt{26}} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}\right) \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$\text{proj}(\vec{e_1}) = \frac{1}{\sqrt{26}} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$\text{proj}(\vec{e_2}) = \frac{3}{\sqrt{26}} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \frac{1}{\sqrt{26}} \begin{bmatrix} 3 \\ 9 \\ 12 \end{bmatrix}$

$\text{proj}(\vec{e_3}) = \frac{4}{\sqrt{26}} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = \frac{1}{\sqrt{26}} \begin{bmatrix} 4 \\ 12 \\ 16 \end{bmatrix}$

$$\boxed{A = \frac{1}{\cancel{\sqrt{26}}\ 26} \begin{bmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ 4 & 12 & 16 \end{bmatrix}}$$

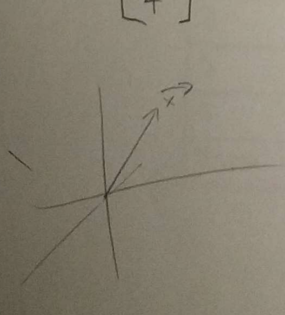
b. (4 points) Find the matrix which represents reflection across the plane in $\mathbb{R}^3$ defined by the equation $x + 3y + 4z = 0$.

3

$\vec{v} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$\text{ref}_v(\vec{x}) = \vec{x} - 2 \cdot \frac{1}{\sqrt{26}} \begin{bmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ 4 & 12 & 16 \end{bmatrix} \vec{x}$

$= \underbrace{\left(I_3 - \frac{2}{\sqrt{26}} \begin{bmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ 4 & 12 & 16 \end{bmatrix} \right)}$ $\vec{x}$



-1

$$\begin{bmatrix} 1 - \frac{2}{\sqrt{26}} & \frac{6}{\sqrt{26}} & \frac{8}{\sqrt{26}} \\ \frac{6}{\sqrt{26}} & 1 - \frac{18}{\sqrt{26}} & \frac{24}{\sqrt{26}} \\ \frac{8}{\sqrt{26}} & \frac{24}{} & 1 - \frac{32}{} \end{bmatrix}$$