Math 33A | Exam #1 | Fall 2014

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By signing here, you confirm you are the person identified above and that all the work herein is solely your own.

Instructions:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers. If you use the back of a page please indicate that you have work on the reverse side.
- You may not use books, notes, calculators, mobile phones, or any outside help during this exam. You may not collaborate with other students in any way during this midterm exam.

Problem	Points	Score
1	10	8
2	10	10
3	10	10
4	10	10
5	10	9
Total	50	47

1. (10 points) For each of the following five statements indicate whether it is true or false. Each part is worth 2 points and there is no penalty for guessing.

(i) If a matrix A is invertible, and $BA = CA$ then $B = C$.
True or **False**

(ii) If a system of linear equations has 5 unknowns (variables) and three equations, then it can have a unique solution.
True or **False**

(iii) The function $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \\ \sqrt{1+x^2+y^2} \end{bmatrix}$ is a linear transformation.
True or **False**

One to one →

$T(a+b) = T(a) + T(b)$

$kT(a) = T(ka)$

(iv) If A and B are matrices such that AB is defined and $AB = I_n$ for some positive integer n, then $BA = I_l$ for some positive integer, l.
True or **False**

l has to equal n

– cant multiply matrices unless rows of first one match rows of other one

– Only way you can multiply AB + BA is $n = l$

(v) If A be an $n \times n$ matrix for which A^2 equals the zero matrix, then $I_n + A$ is invertible.
True or **False**

how to get zero matrix?

can only get zero matrix w/ zero matrix?

2. (10 points)
Use Gauss-Jordan elimination to solve the following system of linear equations. Indicate whether there are zero, one, or infinitely many solutions.

$$x_1 - 2x_2 - x_3 = 1$$
$$2x_1 - 2x_2 - x_3 = 2$$
$$-x_1 + 4x_2 + 2x_3 = 3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 2 & -2 & -1 & 2 \\ -1 & 4 & 2 & 3 \end{array}\right] \begin{array}{l} \\ +2(\text{III}) \\ +\text{I} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 6 & 3 & 8 \\ 0 & 2 & 1 & 4 \end{array}\right] \begin{array}{l} +\text{III} \\ -3\text{III} \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & -4 \\ 0 & 2 & 1 & 4 \end{array}\right]$$

zero solutions

10

3. (10 points)
For the following matrix

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 & 5 \\ 1 & 1 & -1 & 0 & 4 \\ 0 & 2 & -4 & 1 & 3 \end{bmatrix}$$

(II - I) $\to$ $\begin{bmatrix} 1 & 2 & -3 & 0 & 5 \\ 0 & -1 & 2 & 0 & -1 \\ 0 & 2 & -4 & 1 & 3 \end{bmatrix}$ II/-1

(a) (5 points) find rref(A),

$\begin{bmatrix} 1 & 2 & -3 & 0 & 5 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 2 & -4 & 1 & 3 \end{bmatrix}$ -2II, -2(II) $\to$ $\begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$$\boxed{\begin{bmatrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}}$$

5

(b) (5 points) Write down a parameterization for all vectors $\vec{x}$ in $\mathbb{R}^5$ such that $A\vec{x} = \vec{0}$.

$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$

$x_1 = -x_3 - 3x_5$
$x_2 = 2x_3 - x_5$
$x_4 = -x_5$

$x_3 = a$
$x_5 = b$
$x_4 = -b$
$x_1 = -a - 3b$
$x_2 = 2a - b$

5

Check my answer (look below first)

$$AA^{-1} = \begin{bmatrix} 3 & -5/2 & \frac{1}{2} \\ -3 & 4 & -1 \\ 1 & -3/2 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

4. (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}.$$

5

(a) (5 points) Compute A^{-1},

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \\ -\mathrm{I} \\ -\mathrm{I} \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 3 & 8 & -1 & 0 & 1 \end{array}\right] \begin{array}{c} -\mathrm{II} \\ \\ -3\mathrm{II} \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 2 & -3 & 1 \end{array}\right] \begin{array}{c} \\ -\mathrm{III} \\ \mathrm{III}/2 \end{array} \to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 0 & -3 & 4 & -1 \\ 0 & 0 & 1 & 1 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \begin{array}{c} +\mathrm{III} \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -\frac{5}{2} & \frac{1}{2} \\ 0 & 1 & 0 & -3 & 4 & -1 \\ 0 & 0 & 1 & 1 & -3/2 & \frac{1}{2} \end{array}\right]$$

5

(b) (5 points) If possible, solve the system $A\vec{x} = \vec{b}$, with

$$\vec{b} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}.$$

$$\begin{bmatrix} 3 & -\frac{5}{2} & \frac{1}{2} \\ -3 & 4 & -1 \\ 1 & -3/2 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$x_1 = 6 - 10 + 3 = -1 = x_1$

$x_2 = -6 + 16 - 6 = 4 = x_2$

$x_3 = 2 - 6 + 3 = -1 = x_3$

counter-clockwise

5. (10 points) (a) (5 points) Compute the 2×2 matrix which represents rotation by $\pi/4$, followed by reflection across the line $y = x$ in $\mathbb{R}^2$.

← reflection

$\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$

← rotation

$\begin{bmatrix} \cos \pi/4 & -\sin \pi/4 \\ \sin \pi/4 & \cos \pi/4 \end{bmatrix} = \begin{matrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{matrix}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

[rotation] ↓ to rotate

[reflection][rotation] = rotate then reflect

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \boxed{\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}}$

5

(b) (5 points) Compute the matrix which represents orthogonal projection onto the line in the direction of the vector

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{1}{\sqrt{3}}$$

in $\mathbb{R}^3$.

$(u \cdot x)u$

$\left(\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right) \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\frac{1}{3}(x_1 + x_2 + x_3) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$?

$\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

4

How did you