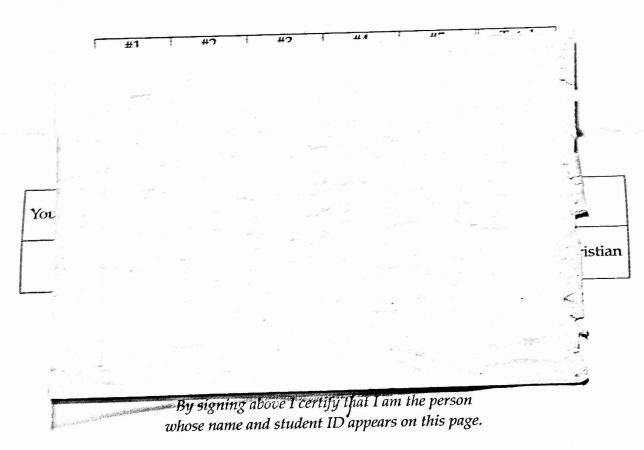
MATH 33A LECTURE 3 2ND MIDTERM

Please note: Show your work. Correct answers not accompanied by sufficent explanations will receive little or no credit. Please call one of the proctors if you have any questions about a problem. Use of calculators, computers, PDAs, cell phones, or other devices is not be permitted during the exam. If you have a question about the grading or believe that a problem has been graded incorrectly, you must bring it to the attention of your professor within 2 weeks of the exam.



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Problem 1. - 1-24238382*--- (True/False, 1 pt each) Mark your answers by filling in the appropriate box next to each question.

(i: (T)(F)) Any basis for \mathbb{R}^3 consists of 3 orthonormal vectors.

(iii. T) If A is a 4×4 orthogonal matrix, $Ae_1 = e_2$, $Ae_2 = e_3$, $Ae_3 = e_1$, then $Ae_4 = e_4$ or $Ae_4 = -e_4$

(iii: T) F) There are no vectors v, w in \mathbb{R}^7 so that $v \cdot w = 7$ and ||v|| = ||w|| = 2.

(iv: T) $(A^t x) \bullet y = x \bullet (Ay)$ for any vectors x, y and any matrix A (here \bullet denotes the dot product)

(v: T) A square orthogonal matrix is invertible.

(vi: T) If $A^{t}A = I$ then A is an orthogonal matrix.

(vif: T(F)) Every basis for the plane x + y + z = 0 consists of two vectors.

viii: T F The columns of an orthogonal matrix are orthonormal.

(ix: T) F) The determinant of an invertible matrix is nonzero.

(x: (T)F) For any matrix A, the kernel of A is perpendicular to the image of A^t .

A-1 = A

Im A

imA

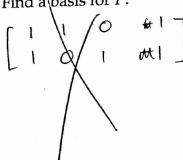
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Problem 2. *--*25414872*--* (10 pts) Let *P* be the subspace of \mathbb{R}^4 consisting of vectors $\begin{bmatrix} y \\ z \\ w \end{bmatrix}$

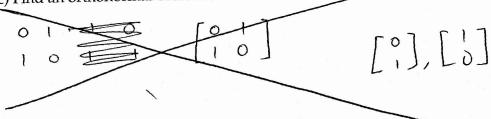
satisfying x + y = -w and x + z = -w.

(a) Find a basis for P.



(b) Find the dimension of P.

(c) Find an orthonormal basis for P.



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4

Problem 3.26137573*... (10 pts) Let

$$A = \left[\begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right].$$

Diagonalize A; in other words, find a basis $\mathfrak B$ in which the matrix A is diagonal.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -2 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

let
$$B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\mathcal{O}\left(\begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}\right)$$

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$AB' = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a+c & 2b+d \\ a+2c & b+2d \end{bmatrix}$$

$$\alpha+2c=2b+d=0$$

 $2a+c=b+2d\neq0$

$$d = -2b$$

$$-3c = -3b + 0$$

$$2a + c \neq 0$$
, $a = 4 - 2$
 $a + 2c = 0$ $c = 2 \cdot 1$
 $2b + d = 0$ $b = 1$
 $b + 2d \neq 0$ $d = -2$.

$$Zb+d=0$$
 $d=-2$



$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

Problem 4.•27532554•...• (10 pts) (a) Suppose that a, b, c are three vectors in \mathbb{R}^4 . Let $p = b(a \bullet c) - c(a \bullet b)$. Show that $p \perp a$.

If
$$p + \alpha$$
, $p \cdot \alpha = 0$.

$$(b(a \cdot c) - c(a \cdot b)) \cdot \alpha \qquad (b(a \cdot c)) \cdot \alpha$$

$$= (ba \cdot bc) \cdot \alpha - (ca \cdot cb) \cdot \alpha$$

$$= (ba \cdot a) \cdot bca - (ca \cdot a) \cdot cb \cdot \alpha = (a \cdot b) (a \cdot c)$$

$$= b \cdot (bca) - c \cdot (acb)$$

$$= ab \cdot (bbc) \cdot (a \cdot b) - (c \cdot a) \cdot (acb)$$

$$= ab \cdot (c \cdot a) \cdot b = 0$$

$$c \cdot (a \cdot b)$$

(b) Suppose that u, v are two vectors in \mathbb{R}^7 so that $u \bullet v = v \bullet v = u \bullet u = 2$. Prove that u = v.

$$u \cdot (u \cdot v)$$

$$||u||^2 ||v||^2 = \langle u, u \rangle \langle v, v \rangle =$$

$$\|u \cdot v\|^2 = \langle u \cdot v, u \cdot v \rangle.$$

Problem 5. (10 pts) Let \mathfrak{B} be the basis for \mathbb{R}^2 given consisting of $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$,

$$v_2 = \left[\begin{array}{c} 1 \\ -1 \end{array} \right].$$

(a) Find the coordinates of the vectors $w_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $w_2 = \begin{bmatrix} 2016 \\ 2017 \end{bmatrix}$ in the basis \mathfrak{B} .

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2$$

$$\begin{bmatrix} W_2 \end{bmatrix}_{B} = \begin{bmatrix} -\frac{1}{2} & \frac{4033}{2} \\ -\frac{1}{2} & \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

(b) Let T be the transformation of \mathbb{R}^2 given by $Te_1 = e_1 + e_2$, $Te_2 = e_1 - e_2$, where e_1, e_2 are the standard basis of \mathbb{R}^2 . Find the matrix of T in the basis \mathfrak{B} .