MATH 33A LECTURE 3
2ND MIDTERM

Please note: Show your work. Correct answers not accompanied by sufficent explanations will receive little or no credit. Please call one of the proctors if you have any questions about a problem. Use of calculators, computers, PDAs, cell phones, or other devices is not be permitted during the exam. *If you have a question about the grading or believe that a problem has been graded incorrectly, you must bring it to the attention of your professor within 2 weeks of the exam.*

By signing above I certify that I am the person whose name and student ID appears on this page.

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Problem 1. *24238382* (True/False, 1 pt each) Mark your answers by filling in the appropriate box next to each question.

(i: T F) Any basis for $\mathbb{R}^3$ consists of 3 orthonormal vectors.

(ii: T F) If A is a 4×4 orthogonal matrix, $Ae_1 = e_2$, $Ae_2 = e_3$, $Ae_3 = e_1$, then $Ae_4 = e_4$ or $Ae_4 = -e_4$

(iii: T F) There are no vectors v, w in $\mathbb{R}^7$ so that $v \cdot w = 7$ and $\|v\| = \|w\| = 2$.

(iv: T F) $(A^t x) \bullet y = x \bullet (Ay)$ for any vectors x, y and any matrix A (here $\bullet$ denotes the dot product)

(v: T F) A square orthogonal matrix is invertible.

(vi: T F) If $A^t A = I$ then A is an orthogonal matrix.

(vii: T F) Every basis for the plane $x + y + z = 0$ consists of two vectors.

(viii: T F) The columns of an orthogonal matrix are orthonormal.

(ix: T F) The determinant of an invertible matrix is nonzero.

(x: T F) For any matrix A, the kernel of A is perpendicular to the image of A^t.

$A^{-1} = A$

im A

im A

Ker CA

im (

Problem 2. *--*25414872*--* (10 pts) Let P be the subspace of $\mathbb{R}^4$ consisting of vectors $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$ satisfying $x + y \overset{+w=0}{=} -w$ and $x + z \overset{+w=0}{=} -w$.

(a) Find a basis for P.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $y = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $z = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

??

(b) Find the dimension of P.

$\dim(P) = 2.$ ✓

$$\begin{bmatrix} 0 & 1 & & \\ 1 & 0 & & \end{bmatrix}$$

(c) Find an orthonormal basis for P.

~~0 1~~
~~1 0~~ ~~$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$~~

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Problem 3. •--•26137573•--• (10 pts) Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

Diagonalize A; in other words, find a basis $\mathfrak{B}$ in which the matrix A is diagonal.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & \\ 1 & \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

2x 2

let $\mathfrak{B} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$$\vec{x} = \left(\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right)$$

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad {}_{B}A_{B'} = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a+c & 2b+d \\ a+2c & b+2d \end{bmatrix}$$

$a+2c = 2b+d = 0$

$2a+c = b+2d \neq 0$

$\cancel{2b = d}$

$d = -2b$

$a = -2c$.

$-3c = -3b \neq 0$.

$2a+c \neq 0$. $a = -2$

$a+2c = 0$ $c = 1$

$2b+d = 0$ $b = 1$

$b+2d \neq 0$ $d = -2$.

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}.$$

hand

Problem 4. *---*27532554*---* (10 pts) (a) Suppose that a, b, c are three vectors in $\mathbb{R}^4$. Let $p = b(a \bullet c) - c(a \bullet b)$. Show that $p \perp a$.

let ~~p~~

$a = $ ~~$\alpha_1 \ \alpha_4$~~ .

if $a = a$,

if $p \perp a$, $p \cdot a = 0$.

$(b(a \cdot c) - c(a \cdot b)) \cdot a$ $\quad (b(a \cdot c)) \cdot a$

No!

$= (ba \cdot bc) \cdot a - (ca \cdot cb) \cdot a$ $\quad = (a \cdot b)(a \cdot c)$

$= (ba \cdot a) \cdot bca - (ca \cdot a) \cdot cb \cdot a$

$= b \cdot (bca) - c \cdot (acb)$

$= \cancel{ac\ ab} \cdot (\cancel{b}\,b\,c) \cdot (a \cdot b) - (c \cdot a) \cdot (\cancel{c}\, c\, b)$

$= \cancel{c(a)b} - (c \cdot a) \cdot b \quad = 0.$

$c \cdot (a \cdot b)$

(b) Suppose that u, v are two vectors in $\mathbb{R}^7$ so that $u \bullet v = v \bullet v = u \bullet u = 2$. Prove that $u = v$.

u, v.

if $u \cdot v = u \cdot u$.

$u \cdot (u \cdot v)$

$\|u\|^2 = \langle u, u \rangle$.

$= \langle u, v \rangle$.

$\|v\|^2 = \langle v, v \rangle$.

$\|u\|^2 \|v\|^2 = \langle u, u \rangle \langle v, v \rangle =$

$\|u \cdot v\|^2 = \langle u \cdot v, u \cdot v \rangle$.

Problem 5. *---*28961149*---* (10 pts) Let $\mathfrak{B}$ be the basis for $\mathbb{R}^2$ given consisting of $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(a) Find the coordinates of the vectors $w_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $w_2 = \begin{bmatrix} 2016 \\ 2017 \end{bmatrix}$ in the basis $\mathfrak{B}$.

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$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ ✓

$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 2016 \\ 2017 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

$\left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}\right)$ $\boxed{[w_1]_{\mathbb{B}} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}}$

$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & -1 & 2 \end{array}\right]$

✓

$\boxed{[w_2]_{\mathbb{B}} = \begin{bmatrix} \frac{4033}{2} \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}}$

$\left[\begin{array}{cc|c} 0 & 2 & 0 \\ 1 & 0 & 2 \end{array}\right]$. $c_2 = 0$, $c_1 = 2$

$\left[\begin{array}{cc|c} 1 & 1 & 2016 \\ 1 & -1 & 2017 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 0 & -2 & 1 \\ 2 & 0 & 4033 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 0 & 1 & -\frac{1}{2} \\ 1 & 0 & \frac{4033}{2} \end{array}\right]$

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(b) Let T be the transformation of $\mathbb{R}^2$ given by $Te_1 = e_1 + e_2$, $Te_2 = e_1 - e_2$, where e_1, e_2 are the standard basis of $\mathbb{R}^2$. Find the matrix of T in the basis $\mathfrak{B}$.

$\vec{x} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \xrightarrow{A} T(\vec{x}) = \begin{bmatrix} e_1+e_2 & e_1-e_2 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\downarrow A\vec{x}$

$[T(\vec{x})]_{\mathbb{B}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e_1 + \begin{bmatrix} \phi \\ 0 \end{bmatrix} e_2 + \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} e_1 - \begin{bmatrix} 1 \\ 0 \end{bmatrix} e_2\right)$

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$[\vec{x}]_{\mathbb{B}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. $\longrightarrow$

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & -1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right] \begin{array}{l} c_1 \\ c_2 \end{array}$

e_1

$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right] - c_1 = 1.$ $\quad \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right]$

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