Math 33A Midterm 1

MATTHEW GUAN

TOTAL POINTS

55 / 55

QUESTION 1

1 Question 1 **10 / 10**

✓ **- 0 pts Everything correct**

- **1 pts** Minor mistakes

- **1.5 pts** Moderate mistakes (such as incorrect rref computation)

- **2.5 pts** Incorrect/Incomplete conclusion

- **0 pts** Something else is incorrect (See the comment)

QUESTION 2

2 Question 2 **10 / 10**

✓ **- 0 pts Correct**

- **1 pts** minor error

- **2 pts** several minor errors

- **3 pts** Incorrect RREF

- **10 pts** Click here to replace this description.

QUESTION 3

3 Question 3 **10 / 10**

✓ **- 0 pts Correct**

- **1 pts** Small computation error

- **3 pts** Incorrect TST

- **3 pts** Incorrect geometric interpretation part (c)

- **10 pts** Blank

QUESTION 4

4 Question 4 **10 / 10**

✓ **- 0 pts (a) Correct: $$T\left[\begin{array}{c} 8 \\ 3\end{array}\right]=\left[\begin{array}{c} 1 \\ 2 \\ 13\end{array}\right]$$**

- **2 pts** (a) Incorrect

✓ **- 0 pts (b) Correct: $$A\left[\begin{array}{cc} 3 & 5 \\ 1 & 2 \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ -2 & 4 \\ 7 & 6\end{array}\right]$$**

- **1 pts** (b) Incorrect/missing explanation

- **2 pts** (b) A little correct work

- **3 pts** (b) Incorrect matrix or didn't compute matrix

✓ **- 0 pts (c) Correct: $$A = \left[\begin{array}{cc} 2 & -5\\ -8 & 22 \\ 8 & -17\end{array}\right]$$**

- **2 pts** (c) Right computation, but incorrect final answer

- **3 pts** (c) Partially correct

- **5 pts** (c) Incorrect

- **1 pts** Small arithmetic mistake

QUESTION 5

5 Question 5 **15 / 15**

Part (a)

✓ **- 0 pts Everything correct**

- **2 pts** Incorrect/incomplete explanation

- **2 pts** Incorrect answer/example

- **3 pts** No solution

- **0 pts** Something else is incorrect (See the comments)

Part (b)

✓ **- 0 pts Everything correct**

- **2 pts** Incorrect/incomplete explanation

- **3 pts** No solution

- **0 pts** Something else is incorrect (See the comments)

Part (c)

✓ **- 0 pts Everything correct**

- **2 pts** Incorrect/incomplete example/explanation

- **3 pts** No solution

- **0 pts** Something else is incorrect (See the comments)

Part (d)

✓ **- 0 pts Everything correct**

- 2 pts Incorrect/incomplete explanation

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)

Part (e)

✓ **- 0 pts Everything correct**

- 2 pts Incorrect/incomplete example/explanation

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)

Math 33A: Midterm 1

2021 Spring

Instructions:

- ☑ The exam will begin on April 19th at 8AM PT. You will be given **24 hours** to complete and submit your works. The submission window will be closed on April 20th at 8AM.
- ☑ **No late submission** will be considered. Make sure to spare enough time to complete and submit your solutions. Make-ups for the exam are permitted only under exceptional circumstances, as outlined in the UCLA student handbook.
- ☑ The exam will be **open book/open notes**. You can use any resources you find in our textbook or on our CCLE page.
- ☑ You must **show your works to receive credit.** Partial credit will be scarce for incomplete solutions, so make sure to get everything right.
- ☑ You may use technology to write up your solutions, such as word processors or note-taking applications. You may also write your solutions on blank papers. If you choose to do so, please leave enough space between questions.
- ☑ A Gradescope link for submitting your work will be provided on the CCLE course webpage.
- ☑ If you have a question about the phrasing of the questions or about the exam logistics, you may email me (sos440@math.ucla.edu). Please make sure to begin the subject line of your email with the prefix 'Math 33A'; otherwise I will not reply to the email.
- ☑ You must **sign the code of conduct.** Any deviation from the rules will be considered as cheating. The university is also well-aware of "academic educational sites", and their use in connection with the exam is an Honor Code violation that is taken very seriously in UCLA.

Please read and sign the following honor code:

> *"I certify, on my honor, that I have not asked for or received assistance of any kind from any other person while working on the exam and that I have not used any non-permitted materials or technologies during the period of this evaluation."*

Name: Matthew Guan

UID: 905502285

Signature: Matt G

Q1. (10 points) Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 8 \\ 3 \\ k \end{bmatrix},$$

where k is a number.

(a) Determine the value of k such that $\vec{v}_3$ is a linear combination of $\vec{v}_1$ and $\vec{v}_2$.

Solve the equation. $\begin{bmatrix} 8 \\ 3 \\ k \end{bmatrix} = x_1 \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

This becomes augmented matrix $\left[\begin{array}{cc|c} 4 & 3 & 8 \\ 1 & 1 & 3 \\ -1 & 1 & k \end{array}\right] = A$. Find rref(A).

$$\to \left[\begin{array}{cc|c} 4 & 3 & 8 \\ 1 & 1 & 3 \\ -1 & 1 & k \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & 3 & 8 \\ -1 & 1 & k \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -1 & -4 \\ 0 & 2 & 3+k \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 4 \\ 0 & 2 & 3+k \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & k-5 \end{array}\right]$$

In order for the system to be consistent, $k-5$ must equal 0, else we get a contradiction. Only when the system has a solution is $\begin{bmatrix} 8 \\ 3 \\ k \end{bmatrix}$ a linear combination of $\vec{v}_1$ and $\vec{v}_2$. Thus, $\boxed{k=5}$ for $\vec{v}_3$ to be a linear combination of $\vec{v}_1$ & $\vec{v}_2$.

(b) Let k be as in the previous part. Find all solutions of the equation

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3$$

and then write your solution in parametric form.

we get the augmented matrix $\left[\begin{array}{ccc|c} 4 & 3 & 8 & 2 \\ 1 & 1 & 3 & 1 \\ -1 & 1 & 5 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 4 & 3 & 8 & 2 \\ -1 & 1 & 5 & 3 \end{array}\right]$.

reduced row echelon form $\to \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & -1 & -4 & -2 \\ 0 & 2 & 8 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & 4 & 2 \\ 0 & 2 & 8 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$

so: $\begin{cases} x_1 - x_3 = -1, \\ x_2 + 4x_3 = 2. \end{cases}$

if we let x_3 be t, $x_1 = t-1$, and $x_2 = 2-4t$.

Thus, solution is $\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t-1 \\ 2-4t \\ t \end{bmatrix} \text{ where } t \in \mathbb{R}.}$

1 Question 1 **10 / 10**

✓ **- 0 pts** **Everything correct**

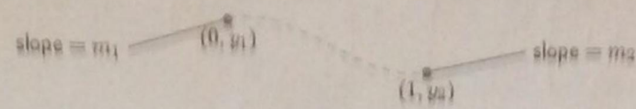
- 1 pts Minor mistakes

- 1.5 pts Moderate mistakes (such as incorrect rref computation)

- 2.5 pts Incorrect/Incomplete conclusion

- 0 pts Something else is incorrect (See the comment)

Q3. (10 points) In computer graphics, we are often interested in finding a curve that interpolates a given set of points in a reasonably smooth way. For instance, consider the situation where we are given two line segments in $\mathbb{R}^2$ with known slopes and endpoints as in the figure below, and suppose we want to find a curve joining them.



One method often employed in this problem is to find a polynomial

$$f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

such that

$$\left| \begin{array}{l} f(0) = y_1 \\ f(1) = y_2 \\ f'(0) = m_1 \\ f'(1) = m_2 \end{array} \right|, \qquad (\diamond)$$

where y_1, y_2, m_1, m_2 are numbers and $f'(t) = a_1 + 2a_2 t + 3a_3 t^2$ is the derivative of $f(t)$.

(a) By regarding a_0, a_1, a_2, a_3 as variables, write down the linear system ($\diamond$) in matrix form.

$f(0) = a_0 = y_1.$
$f(1) = a_0 + a_1 + a_2 + a_3 = y_2.$
$f'(0) = a_1 = m_1$
$f'(1) = a_1 + 2a_2 + 3a_3 = m_2.$

In augmented matrix form, the linear system is

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & y_1 \\ 1 & 1 & 1 & 1 & y_2 \\ 0 & 1 & 0 & 0 & m_1 \\ 0 & 1 & 2 & 3 & m_2 \end{array}\right]$$

(b) Solve the system ($\diamond$) for a_0, a_1, a_2, a_3. In other words, determine the formulas for a_0, a_1, a_2, a_3 in terms of y_1, y_2, m_1, m_2.

Convert to rref.

$$\left[\begin{array}{cccc|c} 1&0&0&0&y_1\\ 1&1&1&1&y_2\\ 0&1&0&0&m_1\\ 0&1&2&3&m_2 \end{array}\right] \to \left[\begin{array}{cccc|c} 1&0&0&0&y_1\\ 0&1&0&0&m_1\\ 1&1&1&1&y_2\\ 0&1&2&3&m_2 \end{array}\right] \to \left(\begin{array}{cccc|c} 1&0&0&0&y_1\\ 0&1&0&0&m_1\\ 0&1&1&1&y_2-y_1\\ 0&1&2&3&m_2 \end{array}\right)$$

$$\to \left[\begin{array}{cccc|c} 1&0&0&0&y_1\\ 0&1&0&0&m_1\\ 0&0&1&1&y_2-y_1-m_1\\ 0&0&2&3&m_2-m_1 \end{array}\right] \to \left[\begin{array}{cccc|c} 1&0&0&0&y_1\\ 0&1&0&0&m_1\\ 0&0&1&1&y_2-y_1-m_1\\ 0&0&0&1&(m_2-m_1)-2(y_2-y_1-m_1) \end{array}\right]$$

$$\to \left[\begin{array}{cccc|c} 1&0&0&0&y_1\\ 0&1&0&0&m_1\\ 0&0&1&0&y_2-y_1-m_1-((m_2-m_1)-2(y_2-y_1-m_1))\\ 0&0&0&1&(m_2-m_1)-2(y_2-y_1-m_1) \end{array}\right]$$

From the rref, we get that

$$\boxed{\begin{aligned} a_0 &= y_1\\ a_1 &= m_1\\ a_2 &= 3y_2-3y_1-2m_1-m_2.\\ a_3 &= m_2+m_1+2y_1-2y_2. \end{aligned}}$$

(c) Find a polynomial $f(t)$ of degree at most 3 such that

$$f(0)=2, \qquad f(1)=2, \qquad f'(0)=3, \qquad f'(1)=-2.$$

We have $y_1=2$, $y_2=2$, $m_1=3$, $m_2=-2$.

Using part (b), we get: $a_0=2.$

$a_1=3.$

$a_2=6-6-6+2=-4.$

$a_3=1+4-4=1.$

$f(t)=a_0+a_1t+a_2t^2+a_3t^3.$

$$\boxed{f(t)=2+3t-4t^2+t^3}$$

2 Question 2 **10 / 10**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** minor error
- **- 2 pts** several minor errors
- **- 3 pts** Incorrect RREF
- **- 10 pts** Click here to replace this description.

Q3. (10 points) Consider the following linear transformations:

- $T : \mathbb{R}^2 \to \mathbb{R}^2$ reflects any vector about the line $y = x$.
- $S : \mathbb{R}^2 \to \mathbb{R}^2$ rotates any vector through an angle of $45°$ in the counter-clockwise direction.

(a) Find the matrices of T and S, respectively.
(That is, find the matrix A such that $T(\vec{x}) = A\vec{x}$ and the matrix B such that $S(\vec{x}) = B\vec{x}$.)

for T, $\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ becomes $\begin{bmatrix} y_1 \\ x_1 \end{bmatrix}$ after reflection

so $A\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} y_1 \\ x_1 \end{bmatrix}$. A is a 2x2 matrix $\to \begin{bmatrix} * & * \\ * & * \end{bmatrix}$.

we see that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. So, matrix of T is $\boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.} = A$.

For matrix B, rotation matrix is $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Since $\theta = \pi/4$, matrix

of S is $\boxed{\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}}$

(b) Find the matrix C of the composition $T \circ S \circ T$.
(That is, find the matrix C such that $(T \circ S \circ T)(\vec{x}) = C\vec{x}$.)

$T(S(T(\vec{x})))$.

let A be matrix of T, B be matrix of S, matrix $C = ABA$.

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

$= \boxed{\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}.}$

(c) You are given that the composition $T \circ S \circ T$ reduces to one of the geometric transformations discussed in class. Interpret the transformation $T \circ S \circ T$ geometrically.

let $\theta = -\pi/4$. So, $\begin{bmatrix} \cos(-\pi/4) & -\sin(-\pi/4) \\ \sin(-\pi/4) & \cos(-\pi/4) \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$.

Geometrically, the tranoformation $T \circ S \circ T$ represents a rotation through an angle $45°$ in the clockwise direction.

3 Question 3 **10 / 10**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** Small computation error
- **- 3 pts** Incorrect TST
- **- 3 pts** Incorrect geometric interpretation part (c)
- **- 10 pts** Blank

Q4. (10 points) Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ such that

$$T\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\7\end{bmatrix} \quad \text{and} \quad T\begin{bmatrix}5\\2\end{bmatrix} = \begin{bmatrix}0\\4\\6\end{bmatrix}.$$

Let A be the matrix of T, i.e., A is the matrix such that $T(\vec{x}) = A\vec{x}$.

(a) Compute $T\begin{bmatrix}8\\3\end{bmatrix}$. $\begin{bmatrix}8\\3\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix} + \begin{bmatrix}5\\2\end{bmatrix}$.

By linearity, $T\left(\begin{bmatrix}8\\3\end{bmatrix}\right) = T\left(\begin{bmatrix}3\\1\end{bmatrix}\right) + T\left(\begin{bmatrix}5\\2\end{bmatrix}\right)$.

$T\left(\begin{bmatrix}8\\3\end{bmatrix}\right) = \begin{bmatrix}1\\-2\\7\end{bmatrix} + \begin{bmatrix}0\\4\\6\end{bmatrix} = \boxed{\begin{bmatrix}1\\2\\13\end{bmatrix}}$.

(b) What is the matrix product $A\begin{bmatrix}3 & 5\\1 & 2\end{bmatrix}$? Briefly explain why. $A\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\7\end{bmatrix}$, and

A is a 3x2 matrix. We have $A\begin{bmatrix}5\\2\end{bmatrix}$ is equal to $\begin{bmatrix}0\\4\\6\end{bmatrix}$. The columns of $\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$ are represented by $\begin{bmatrix}3\\1\end{bmatrix}$ and $\begin{bmatrix}5\\2\end{bmatrix}$, two column vectors. By Thm. 2.3.2 in the textbook, $A\begin{bmatrix}3 & 5\\1 & 2\end{bmatrix} = \left[A\begin{bmatrix}3\\1\end{bmatrix} \; A\begin{bmatrix}5\\2\end{bmatrix}\right]$

$= \boxed{\begin{bmatrix}1 & 0\\-2 & 4\\7 & 6\end{bmatrix}}$.

The reason why $A\begin{bmatrix}3 & 5\\1 & 2\end{bmatrix} = \begin{bmatrix}1 & 0\\-2 & 4\\7 & 6\end{bmatrix}$ is because of Thm. 2.3.2 in textbook, which states that we can multiply A by columns of $\begin{bmatrix}3 & 5\\1 & 2\end{bmatrix}$, which are $\begin{bmatrix}3\\1\end{bmatrix}$ and $\begin{bmatrix}5\\2\end{bmatrix}$, and combine the resulting vectors into a single matrix. since we know $A\begin{bmatrix}3\\1\end{bmatrix}$ and $A\begin{bmatrix}5\\2\end{bmatrix}$ from the problem statement, we can combine the two resulting 3x1 column vectors

4. c). We have $T\left(\begin{bmatrix}3\\1\end{bmatrix}\right)=\begin{bmatrix}1\\-2\\7\end{bmatrix}$.

and $T\left(\begin{bmatrix}5\\2\end{bmatrix}\right)=\begin{bmatrix}0\\4\\6\end{bmatrix}$. By linearity properties, we find $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.

so $T\left(\begin{bmatrix}15\\5\end{bmatrix}\right)=5\begin{bmatrix}1\\-2\\7\end{bmatrix}=\begin{bmatrix}5\\-10\\35\end{bmatrix}$.

$T\left(\begin{bmatrix}15\\6\end{bmatrix}\right)=3\begin{bmatrix}0\\4\\6\end{bmatrix}=\begin{bmatrix}0\\12\\18\end{bmatrix}$.

so $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=T\left(\begin{bmatrix}15\\6\end{bmatrix}\right)-T\left(\begin{bmatrix}15\\5\end{bmatrix}\right)=\begin{bmatrix}-5\\22\\-17\end{bmatrix}$.

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similarly, $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=T\left(\begin{bmatrix}6\\2\end{bmatrix}\right)-T\left(\begin{bmatrix}5\\2\end{bmatrix}\right)$

$$=2T\begin{bmatrix}3\\1\end{bmatrix}-T\begin{bmatrix}5\\2\end{bmatrix}=\begin{bmatrix}2\\-4\\14\end{bmatrix}-\begin{bmatrix}0\\4\\6\end{bmatrix}$$

$$=\begin{bmatrix}2\\-8\\8\end{bmatrix}$$

Using same logic as ~~in part (a)~~, we get that the matrix A is $\boxed{\begin{bmatrix}2 & -5\\-8 & 22\\8 & -17\end{bmatrix}}$.

4 Question 4 **10 / 10**

✓ **- 0 pts** **(a) Correct: $$T\left[\begin{array}{c} 8 \\ 3\end{array}\right]=\left[\begin{array}{c} 1 \\ 2 \\ 13\end{array}\right]$$**

- 2 pts (a) Incorrect

✓ **- 0 pts** **(b) Correct: $$A\left[\begin{array}{cc} 3 & 5 \\ 1 & 2 \end{array}\right] = \left[\begin{array}{cc} 1 & 0 \\ -2 & 4 \\ 7 & 6\end{array}\right]$$**

- 1 pts (b) Incorrect/missing explanation

- 2 pts (b) A little correct work

- 3 pts (b) Incorrect matrix or didn't compute matrix

✓ **- 0 pts** **(c) Correct: $$A = \left[\begin{array}{cc} 2 & -5\\ -8 & 22 \\ 8 & -17\end{array}\right]$$**

- 2 pts (c) Right computation, but incorrect final answer

- 3 pts (c) Partially correct

- 5 pts (c) Incorrect

- 1 pts Small arithmetic mistake

Q5. (15 points) For each of the following problems, determine whether a matrix with the given properties exists or not. If such a matrix exists, then provide an example and briefly explain why your choice of matrix has the desired properties. Otherwise, use the results from class to explain why there is no such matrix.

(a) A 3×2 matrix A such that the system $A\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ has a unique solution. $\boxed{\text{Such matrix exists}}$

Unique sol. means rank $= m = 2$.
Such matrix exists and let A be $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ The rank of A equals $2 = m$, so we have a unique solution. The last row becomes $0 = 0$, which is not a contradiction.

So, a matrix exists with the following description and an example is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

(b) A 2×2 matrix A such that $A \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$. $\boxed{\text{No such matrix exists.}}$

let A be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, so $A \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} a+2b & 3a+6b \\ c+2d & 3c+6d \end{bmatrix}$ $= \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$, we get the following system of equations.

$\begin{cases} a+2b=1 \\ 3a+6b=-1 \\ c+2d=2 \\ 3c+6d=-2 \end{cases}$

we get a contradiction as if $(a+2b)=1$, $3a+6b$ can't possibly equal -1, as $3a+6b = 3(a+2b) = 3(1) \neq -1$.
thus, there is no such matrix A such that $A \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$.

(c) A 2×2 matrix A such that $A \neq I_2$ but $A^5 = I_2$.
(Here, $A^5 = AAAAA$ denotes the product of five A's, and I_n is the $n \times n$ identity matrix.)

$\boxed{\text{Such matrix exists}}$ we want to find what transformation, repeated five times, is equal to an identity transformation. we see that if we rotate an object $72°$ ~~clockwise~~ counter clockwise five times, this is equal to a $360°$ ccw rotation. So, we let A be the rotation matrix $\begin{bmatrix} \cos(72°) & -\sin(72°) \\ \sin(72°) & \cos(72°) \end{bmatrix}$, then $A^5 = I_2$. So such matrix A exists and is equal to $\begin{bmatrix} \cos(72°) & -\sin(72°) \\ \sin(72°) & \cos(72°) \end{bmatrix}$.

(d) An invertible 2×2 matrix A such that $\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} A$ is also invertible. $\boxed{\text{No such matrix exists}}$

let A be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ for unknown constants a, b, c, d.

$$\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+3c & b+3d \\ 3a+9c & 3b+9d \end{bmatrix}.$$

Determinant $= 3(b+3d)(a+3c) - (b+3d)\cdot 3 \cdot (a+3c) = 0.$

regardless of A, since the determinant always equals 0

no such A exists such that $\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} A$ is also invertible.

(e) A 2×2 matrix A such that $kI_2 - A$ is invertible for all k in $\mathbb{R}$.

$\boxed{\text{Such matrix exists}}$ Let A be the matrix $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$. Thus, $kI_2 - A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} k-2 & 1 \\ -1 & k-2 \end{bmatrix}$.

$\begin{bmatrix} k-2 & 1 \\ -1 & k-2 \end{bmatrix}$ is invertible when $\det(kI_2 - A) \neq 0$. so we have the equation $(k-2)^2 + 1 = 0$.

$(k-2)^2 = -1$, → this equation has no real solution, So, for no value of k is ~~this~~ the determinant $\begin{bmatrix} k-2 & 1 \\ -1 & k-2 \end{bmatrix}$ equal to 0, and $\begin{bmatrix} k-2 & 1 \\ -1 & k-2 \end{bmatrix}$ is always invertible for all k in $\mathbb{R}$. Hence,

if we let A be $\boxed{\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}}$, we get that $kI_2 - A$ is invertible for all k.

5 Question 5 15 / 15

Part (a)

✓ **- 0 pts** **Everything correct**

- 2 pts Incorrect/incomplete explanation

- 2 pts Incorrect answer/example

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)

Part (b)

✓ **- 0 pts** **Everything correct**

- 2 pts Incorrect/incomplete explanation

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)

Part (c)

✓ **- 0 pts** **Everything correct**

- 2 pts Incorrect/incomplete example/explanation

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)

Part (d)

✓ **- 0 pts** **Everything correct**

- 2 pts Incorrect/incomplete explanation

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)

Part (e)

✓ **- 0 pts** **Everything correct**

- 2 pts Incorrect/incomplete example/explanation

- 3 pts No solution

- 0 pts Something else is incorrect (See the comments)