Instructions:

- This exam is in two parts.
  - Part 1 is on CCLE- This is a multiple choice "quiz" that will be graded by the computer, similar to our pre-class assignments.
  - Part 2 is in the following pages and should be written by hand and uploaded to Gradescope.
- You should complete the exam and submit by 8am on Nov 21st PST. Please leave enough time to scan your work into a PDF and upload it to Gradescope! Exams will not be accepted after 8am.
- You may spend as much time as you like on this exam between now and 8am on Nov 21st.
- If you do not have a way to print this exam you can copy each question onto a blank piece of paper. BEWARE- Please copy the entire question, and give yourself plenty of space to answer each question. We will take off points if, for example, your answer says

  2) [your answer]

  without including the phrasing of the question 2.

  Using an ipad or tablet to write directly on the exam is also acceptable.
- This exam is open book. You can use any resources you find in our textbook, on our CCLE page or on the internet in general. However you should not have anyone's help to do the exam. So you are not allowed to ask your classmates or TAs about the questions or to post the exam questions on an online forum. Posting our exam questions online is the same asking someone to do the exam for you which is cheating.
- If you have a question about the phrasing of the one of the questions or about the mechanics of completing the exam, you can email ProfRosesMathExamQuestions@gmail.com.

By signing below, you certify that the following exam (including this written part and the part on CCLE) is entirely your own work and that you did not receive any help in completing the exam. You will not post the exam questions on public or class forums or discuss the questions with anyone else until after the exam has ended.

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(Signature)

Remember to explain your calculations for full credit!

OVER →

1. (6 pts) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 6 & 3 \\ 3 & 6 & 9 & -1 \end{bmatrix}$$

(a) Find a basis for Im(A). Be sure to show your work.

(b) Find basis for ker(A). Be sure to show your work.

(c) Give a geometrical description of the spaces you found in parts (a) and (b).

The image of A is a __________ (line/plane/other space) in __________ ($\mathbb{R}^n$ for what n)

The kernel of A is a __________ (line/plane/other space) in __________ ($\mathbb{R}^n$ for what n)

OVER $\rightarrow$

2. (6 points) For each of the following, given an example of a matrix with the described property.

   (a) A 3×3 nonzero skew-symmetric matrix.

   (b) A matrix that is similar to the matrix A below, but not equal to it.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

   (c) A matrix representing an orthogonal transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(e_1) = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$

OVER $\rightarrow$

3. (6 pts)

(a) Use the Gram Schmidt algorithm to find an orthonormal basis for the plane V, spanned by the vectors $(2, 0, 0)$ and $(5, 4, -3)$

(b) Compute the orthogonal projection of the standard basis vector e_2 onto the plane V.

(c) Find a lower bound for the length of a vector $e_2 - \bar{v}$ for any vector $\bar{v}$ in V. In other words, find a value of k such that $\|e_2 - v\| \geq k$ for all vectors $v \in V$.

OVER $\rightarrow$

4. (6 points) Suppose that $\bar{u}$ and $\bar{v}$ are linearly independent vectors in $\mathbb{R}^2$

(a) Can you guarantee that the set $\{\bar{u}, \bar{u}+\bar{v}\}$ is linearly independent? Why or Why not. Either give a reason why this must be true or give an example of $\bar{u}, \bar{v}$ that shows this is false.

(b) Now suppose that A is a 2×2 nonzero matrix. Can you guarantee that the set $\{A\bar{u}, A\bar{v}\}$ is linearly independent? Why or Why not. Either give a reason why this must be true or give an example of $\bar{u}, \bar{v}$ and A that shows this is false