1. (6 pts) Consider a quadratic polynomial (a function of the form $f(x) = ax^2 + bx + c$) which intersects the points (1,-2), (-1,-6), (-2,-11) and (2,-3). Does such a polynomial exist? If so, find all possible values for a, b and c. Be sure to show the steps of your calculation including any row reductions.

OVER $\rightarrow$

2. (5 pts) Consider the following linear transformations:

$T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, -x_1 - x_2 - x_3, x_1 + x_2 + x_3)$

$S : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates by $90°$ about the z-axis (counterclockwise as viewed from the positive z-axis).

(a) Find a matrix A such that $T(\bar{x}) = A\bar{x}$ and a matrix B such that $S(\bar{x}) = B\bar{x}$

(b) Compute a matrix C such that composition $S \circ T(\bar{x}) = C\bar{x}$.

(c) Is the matrix C invertible? Why or Why not?

OVER →

3. (5) Consider the following matrix.

$$\begin{bmatrix} a & 2 & 0 & 0 \\ 0 & b & c & 1 \\ d & 0 & 0 & 0 \end{bmatrix}$$

If we know this matrix is a rref, can you determine the values of a, b, c and d? For each of these variables either

i) determine what value that variable must take and justify this

or

ii) give two distinct rrefs with different values for the given variable.

OVER $\rightarrow$

4. (12 pts) Give examples with each of the following properties, and explain briefly (1-2 sentences), why the example you chose has the desired property. If no such example exists, use one of our theorems from class to explain why this can't happen.

   (a) An example of a 4x3 matrix A where $A\bar{x} = \bar{0}$ has a unique solution.

   (b) An example of a system of linear equations that does not have infinitely many solutions, but where the rank of the coefficient matrix is less than the number of variables.

   (c) An example of matrix B where $B^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

OVER →

(d) An example of a vector $\bar{v}$ in $\mathbb{R}^3$, which is not a linear combination of the vectors $(1, 1, 0)$ and $(0, 0, 1)$

(e) An example of two vectors $\bar{v}_1$ and $\bar{v}_2$ such that the set of all linear combinations of $\bar{v}_1$ and $\bar{v}_2$ is equal to the vectors on the line $y = -2x$.

(f) An example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(e_1) = (1, 2, 3)$, $T(e_2) = (-2, -4, -6)$ and $T(1, 1) = (0, 0, 0)$