Spring 2021 Math 33A Exam 1

Instructions:

- You should complete the exam and submit by 8am on April 20th PDT. Please leave enough time to scan your work into a PDF and upload it to Gradescope! Exams will not be accepted after 8am.
- You may spend as much time as you like on this exam between now and the due date.
- If you do not have a way to print this exam you can copy each question onto a blank piece of paper. Please copy the question, and give yourself plenty of space to answer each question.
- This exam is open book. You can use any resources you find in our textbook, on our CCLE page or on the internet in general. However you should not have anyone's help to do the exam. So you are not allowed to ask your classmates or TAs about the questions or to post the exam questions on an online forum. Posting our exam questions online is the same asking someone to do the exam for you which is cheating.
- If you have a question about the phrasing of the one of the questions or about the mechanics of completing the exam, you can email ProfRosesMathExamQuestions@gmail.com.

By signing below, you certify that the following exam is entirely your own work and that you did not receive any help in completing the exam. You will not post the exam questions on public or class forums or discuss the questions with anyone else until after the exam has ended.

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(Signature)

Remember to explain your calculations for full credit! It's fine to use technology to check your answer, but please write out some intermediate steps in your row reductions so we can see your process!

OVER →

1. (6 pts) Consider a quadratic polynomial (a function of the form $f(x) = ax^2 + bx + c$) which intersects the points (1,-2), (-1,-6), (-2,-11) and (2,-3). Does such a polynomial exist? If so, find all possible values for a, b and c. Be sure to show the steps of your calculation including any row reductions.

OVER →

2. (5 pts) Consider the following linear transformations:

$T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, -x_1 - x_2 - x_3, x_1 + x_2 + x_3)$

$S : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates by $90°$ about the z-axis (counterclockwise as viewed from the positive z-axis).

(a) Find a matrix A such that $T(\bar{x}) = A\bar{x}$ and a matrix B such that $S(\bar{x}) = B\bar{x}$

(b) Compute a matrix C such that composition $S \circ T(\bar{x}) = C\bar{x}$.

(c) Is the matrix C invertible? Why or Why not?

OVER →

3. (5) Consider the following matrix.

$$\begin{bmatrix} a & 2 & 0 & 0 \\ 0 & b & c & 1 \\ d & 0 & 0 & 0 \end{bmatrix}$$

If we know this matrix is a rref, can you determine the values of a, b, c and d? For each of these variables either

i) determine what value that variable must take and justify this

or

ii) give two distinct rrefs with different values for the given variable.

OVER →

4. (12 pts) Give examples with each of the following properties, and explain briefly (1-2 sentences), why the example you chose has the desired property. If no such example exists, use one of our theorems from class to explain why this can't happen.

   (a) An example of a 4x3 matrix A where $A\bar{x} = \bar{0}$ has a unique solution.

   (b) An example of a system of linear equations that does not have infinitely many solutions, but where the rank of the coefficient matrix is less than the number of variables.

   (c) An example of matrix B where $B^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

OVER →

(d) An example of a vector $\bar{v}$ in $\mathbb{R}^3$, which is not a linear combination of the vectors $(1, 1, 0)$ and $(0, 0, 1)$

(e) An example of two vectors $\bar{v}_1$ and $\bar{v}_2$ such that the set of all linear combinations of $\bar{v}_1$ and $\bar{v}_2$ is equal to the vectors on the line $y = -2x$.

(f) An example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(e_1) = (1, 2, 3)$, $T(e_2) = (-2, -4, -6)$ and $T(1, 1) = (0, 0, 0)$