Instructions:

- This exam is in two parts.
  - Part 1 is on CCLE- This is a multiple choice "quiz" that will be graded by the computer, similar to our pre-class assignments.
  - Part 2 is in the following pages and should be written by hand and uploaded to Gradescope.
- You should complete the exam and submit by 8am on Oct 31st PDT. Please leave enough time to scan your work into a PDF and upload it to Gradescope! Exams will not be accepted after 8am.
- You may spend as much time as you like on this exam between now and 8am on Oct 31st.
- If you do not have a way to print this exam you can copy each question onto a blank piece of paper. BEWARE- Please copy the entire question, and give yourself plenty of space to answer each question. We will take off points if, for example, your answer says

  2) [your answer]

  without including the phrasing of the question 2.
- This exam is open book. You can use any resources you find in our textbook, on our CCLE page or on the internet in general. However you should not have anyone's help to do the exam. So you are not allowed to ask your classmates or TAs about the questions or to post the exam questions on an online forum. Posting our exam questions online is the same asking someone to do the exam for you which is cheating.
- If you have a question about the phrasing of the one of the questions or about the mechanics of completing the exam, you can email ProfRosesMathExamQuestions@gmail.com.

By signing below, you certify that the following is exam (including this written part and the part on CCLE) is entirely your own work and that you did not receive any help in completing the exam. You will not post the exam questions on public or class forums or discuss the questions with anyone else until after the exam has ended.

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(Signature)

Remember to explain you calculations for full credit! It's fine to use technology to check your answer, but please write out some intermediate steps in your row reductions so we can see your process!

OVER →

1. (6 pts) Consider the following linear transformations:

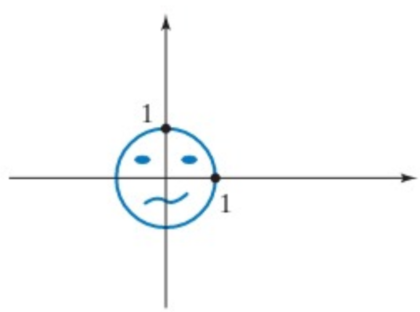
   $T : \mathbb{R}^2 \to \mathbb{R}^2$ scales the x axis by 3 and the y axis by $\frac{1}{2}$.

   $S : \mathbb{R}^2 \to \mathbb{R}^2$ reflects about the line $y = x$

   (a) Find a matrix A such that $T(\bar{x}) = A\bar{x}$ and a matrix B such that $S(\bar{x}) = B\bar{x}$

   (b) Compute a matrix C such that the composition $T \circ S$ maps the vector $\bar{x}$ to the vector $C\bar{x}$.

   (c) Given the picture below, draw what this picture looks like after it has undergone the transformation $T \circ S$



   (d) Is the transformation $T \circ S$ invertible? Why or Why not?

OVER →

2. (6 pts) Consider the vectors

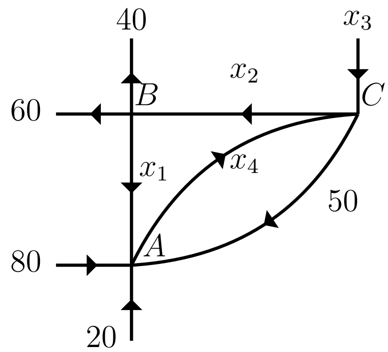
$$\bar{v}_1 = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix} \quad \bar{v}_2 = \begin{bmatrix} 4 \\ -1 \\ -6 \end{bmatrix} \quad \bar{v}_3 = \begin{bmatrix} 6 \\ 4 \\ h \end{bmatrix}$$

(a) For what values of h and k is the vector $\bar{v} = \begin{bmatrix} -4 \\ k \\ 0 \end{bmatrix}$ a linear combination of $\bar{v}_1, \bar{v}_2, \bar{v}_3$? Be sure to explain how you got your answer by showing the steps of your calculation!

(b) For what values of h and k will there be infinitely many possible ways to write $\bar{v}$ as a linear combination of $\bar{v}_1, \bar{v}_2, \bar{v}_3$?

OVER →

3. (6 pts) Consider the following network of streets. The nodes A, B and C represent intersections. Each directed edge between two nodes represents a one-way street and the number associated with the edge represents the number of cars on this street. For example there are 50 cars on the road going from C to A.



Each intersection must have an equal number of ingoing and outgoing cars. Write a system of linear equations that represents the traffic flow on this system of roads, and find a parametric description of the solution set for this system of equations.

OVER →

4. (6 pts) Draw a matrix with each of the following properties, and explain briefly (1-2 sentences), why the matrix you chose has the desired property. If no such matrix exists, use one of our theorems from class to explain why this can't happen.

   (a) An invertible matrix where the third row is a linear combination of the first two rows.

   (b) A matrix A, where $A \neq I$ and $A^3 = A$

   (c) A matrix A representing a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$, where A has at least one non-zero entry, but A maps infinitely many vectors in $\mathbb{R}^3$ to the vector $\bar{0}$ in $\mathbb{R}^4$.