Instructions:

- You should complete the exam and submit by 8am on June 7 PST. Please leave enough time to scan your work into a PDF and upload it to Gradescope! Exams will not be accepted after 8am.
- You may spend as much time as you like on this exam between now and the due date.
- If you do not have a way to print this exam or write directly on the PDF using a tablet, please copy each question onto a blank piece of paper. You final submission should include, the questions, your answers and any scratch work.
- This exam is open book. You can use any resources you find in our textbook, on our CCLE page or on the internet in general. However you should not have anyone's help to do the exam. So you are not allowed to ask your classmates or TAs about the questions or to post the exam questions on an online forum. Posting our exam questions online is the same asking someone to do the exam for you which is cheating.
- If you have a question about the phrasing of the one of the questions or about the mechanics of completing the exam, you can email ProfRosesMathExamQuestions@gmail.com.

SCORE: 49/50, missed 7B.

By signing below, I certify that the following exam is entirely my own work. I did not receive any help in completing the exam. I will not post the exam questions on public or class forums or discuss the questions with anyone else until after the exam has ended.

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(Signature)

Remember to explain your calculations for full credit! It's fine to use technology to check your answer, but please write out some intermediate steps in your row reductions so we can see your process!

OVER →

1. (7 pts) Suppose that the vector $\bar{v} = (a, -2)$ and the vector $\bar{w} = (2a, 9)$ are orthogonal vectors.

Also suppose that the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & a & 1 \\ 0 & 6 & a-1 \end{bmatrix}$$

is invertible.

(a) Use this information to solve for a.

$v \cdot w = 0$

$a \cdot 2a + -2 + 9 = 0$

$2a^2 + -18 = 0$

$a = \pm 3$

if A is invertible

for $a = 3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & a & 1 & 0 & 1 & 0 \\ 0 & 6 & a-1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 6 & 2 \end{bmatrix} \Rightarrow \det(A) \text{ must } \neq 0.$$

$\det(A) = 1 \cdot \det\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix} - 1 \det\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix} + 1 \det\begin{pmatrix} 0 & 3 \\ 0 & 6 \end{pmatrix}$

$= 1 \cdot 0 - 1 \cdot 0 + 1 \cdot 0$

$= 0 \quad \therefore \quad a \neq 3.$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 6 & -4 \end{bmatrix}$$

det(A) must not be 0.

$\det(A) = 1 \cdot \det\begin{pmatrix} -3 & 1 \\ 6 & -4 \end{pmatrix} - 1 \det\begin{pmatrix} 0 & 1 \\ 0 & -4 \end{pmatrix} + 1 \det\begin{pmatrix} 0 & -3 \\ 0 & 6 \end{pmatrix}$

$= 1 \cdot 6 - 1 \cdot 0 + 1 \cdot 0$

$= 6 \quad \therefore \quad a = -3.$

(b) Use your answer from part a) to compute the matrix A^{-1}. (Be sure to show your work)

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 & 1 & 0 \\ 0 & 6 & -4 & 0 & 0 & 1 \end{array}\right] \xRightarrow{R_3 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 2 & 1 \end{array}\right] \xRightarrow{R_3/-2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1/2 \end{array}\right] \xRightarrow{\frac{(R_2 - R_3)}{-3}} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{-3} & -1/6 \\ 0 & 0 & 1 & 0 & -1 & -1/2 \end{array}\right] \xRightarrow{R_1 - R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & +5/3 & +2/3 \\ 0 & 1 & 0 & 0 & -2/3 & -1/6 \\ 0 & 0 & 1 & 0 & -1 & -1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 5/3 & 2/3 \\ 0 & -2/3 & -1/6 \\ 0 & -1 & -1/2 \end{bmatrix}$$

(c) Use your answer from part a) to compute the determinant of the matrix 2A.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 6 & -4 \end{bmatrix} \therefore 2A = \begin{bmatrix} 2 & 2 & 2 \\ 0 & -6 & 2 \\ 0 & 12 & -8 \end{bmatrix}$$

$\det(2A) = 2 \cdot \underbrace{\det\begin{pmatrix} -6 & 2 \\ 12 & -8 \end{pmatrix}}_{\substack{48-24 \\ 24}} - 2 \underbrace{\det\begin{pmatrix} 0 & 2 \\ 0 & -8 \end{pmatrix}}_{0-0} + 2 \underbrace{\det\begin{pmatrix} 0 & -6 \\ 0 & 12 \end{pmatrix}}_{0-0}$

$= 2 \cdot 24 - 2 \cdot 0 + 2 \cdot 0$

$\det 2A = 48$

OVER →

2. (6pts) (Note: you may use a calculator or other technology to check for errors in the problem, but you should be sure to show your work, so I can verify that you didn't exclusively use a calculator.)

Consider the matrix

$(3\times 3)(3\times 2)(2\times 2)$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$A = 3\times 2$

$= 3\times 2$

(a) Compute the singular values for A.

$$A^TA = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$$

$$(3-\lambda)(3-\lambda) = (\lambda^2 - 6\lambda)$$

$$\lambda(\lambda-6)$$

$$\lambda = 0, 6$$

singular values $= \sqrt{6}, 0$

(b) Compute the SVD for A. In other words, compute orthogonal matrices U and V and a matrix Σ such that $A = U\Sigma V^T$.

$A^TA = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

$(3-\lambda)(3-\lambda) = (\lambda^2-6\lambda)$

$\lambda(\lambda-6)$

$\lambda = 0, 6$

for $\lambda = 6$

v_1: $\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$

$x_1 - x_2 = 0$

$\therefore x_1 = x_2$

let $x_2 = 1$ $\therefore v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$v_2 = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$x_1 = -x_2$

let $x_2 = 1$ $\therefore v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\underset{\Sigma}{S} = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\left.\begin{matrix} v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda_6 & \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \\ v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \lambda_0 & \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \end{matrix}\right\} V$$

$U = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$U_1 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \end{bmatrix}$

orthogonal vectors

$\begin{matrix} 1 \\ 1 \\ -1 \end{matrix}$ to $\begin{matrix} 1 \\ 0 \\ 1 \end{matrix}$ to $\begin{matrix} -1 \\ 1 \\ 0 \end{matrix}$

$$U = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\ 1/\sqrt{3} & 0 & \sqrt{2/3} \\ -1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

$\lambda_1 = 6$ $\quad v_1 = \text{span}\begin{bmatrix} +1 \\ +1 \\ -1 \end{bmatrix} = \|v_1\| = \sqrt{3} \Rightarrow \begin{bmatrix} +1/\sqrt{3} \\ +1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$

$\lambda_2 = 0$ $\quad v_2 = \text{span}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \|v_2\| = \sqrt{2} \Rightarrow \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$

$v_3 = \text{span}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \|v_3\| = \sqrt{2} \Rightarrow \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$

G schmidt:

$e_1 = \frac{u_1}{\|u_1\|} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$

$U = \begin{bmatrix} +1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{2} \\ +1/\sqrt{3} & +1/\sqrt{2} & 0 \\ -1/\sqrt{3} & 0 & 1/\sqrt{2} \end{bmatrix}$

$e_2 = \frac{u_2}{\|u_2\|}$ $\quad u_2 = v_2 - \text{proj}_{u_1} v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$e_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

$e_3 = \frac{u_3}{\|u_3\|}$

$u_3 = v_3 - \text{proj}_{u_2} v_3 - \text{proj}_{u_1} v_3 = \begin{bmatrix} -1/2 \\ 1 \\ 1/2 \end{bmatrix}$

$e_3 = \begin{matrix} -1/\sqrt{6} \\ \sqrt{2/3} \\ 1/\sqrt{6} \end{matrix}$

Now....

$$\Sigma = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \quad V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \ (m\times m)$$

OVER →

$$A = U\Sigma V^T = \begin{bmatrix} +1/\sqrt{3} & 1/\sqrt{2} & -1/\sqrt{6} \\ +1/\sqrt{3} & 0 & \sqrt{2/3} \\ -1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

3. (5 pts) Consider the following matrix A, which defines a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(\bar{x}) = A\bar{x}$.

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

(a) Given an independent list of vectors u, v in $\mathbb{R}^2$, is the list $T(\bar{u}), T(\bar{v})$ also independent. Justify your answer.

Yes. $T(x) = A\bar{x}$ is a one to one linear transformation, thus linearity is preserved

$\ker(A) = 0$ ∴ system has a unique solution for each

(b) Given orthogonal vectors u, v in $\mathbb{R}^2$, are the vectors $T(\bar{u}), T(\bar{v})$ also orthogonal. Justify your answer.

No. $T(x) = A\bar{x}$ can change orthogonality by changing the x component and $A^TA = I_2$ thus orthogonality is not preserved

Justification:

$A^TA = I_2 = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$2 \cdot 1 + 3 \cdot 0 = 2$

(c) If R is a region of the plane with area equal to 4 units^2, what is the area of the image of this region after it undergoes the transformation T?

$\operatorname{im}(R) = 4 \text{ units}^2$,

Using a square,

$\overset{v_1}{\begin{bmatrix} 2 \\ 0 \end{bmatrix}} \times \overset{v_2}{\begin{bmatrix} 0 \\ 2 \end{bmatrix}} = 4$

$\overset{Av_1}{\begin{bmatrix} 4 \\ 0 \end{bmatrix}} \times \overset{Av_2}{\begin{bmatrix} 2 \\ 6 \end{bmatrix}} = 24 \text{ units}^2$

also using the $\det(A) = 6 \ldots$

area $\cdot \det(A) = 6 \cdot$ area

$= 24 \text{ units}^2$

OVER →

4. (4 pts) Find an example of a matrix that is invertible and has at least one eigenvalue, but is not diagonalizable. Be sure to explain why your given matrix has the desired properties.

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ 2 linearly independent columns $\therefore$

$\det(A) = 1 - 0 = 1$ $\therefore$ invertible

Has an eigenvalue of

$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - 0$

$\lambda = 1$, almu 2.

$\begin{bmatrix} 1-1 & 1 \\ 0 & 0 \end{bmatrix}$ thus, eigenvector $= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

However, since it only has one linearly independent eigenvector, not 2+, it is not diagonalizable.

OVER →

5. (8 pts) For each of the following descriptions of a square matrix below, identify it as either an invertible matrix, or not an invertible matrix. Justify your answer using a theorem that we have learned in our class.

(a) A 3×3 matrix whose rows are the vectors $\bar{v}, \bar{u}$ and $\bar{v} - \bar{u}$

Sum 7.1.5

$v = (6 \ 2 \ 6)$

$u = (4 \ 0 \ 4)$

$v - u = (2 \ 2 \ 2)$

$$\begin{bmatrix} 6 & 2 & 6 \\ 4 & 0 & 4 \\ 2 & 2 & 2 \end{bmatrix}$$

same columns

it is not linearly independent, and is not invertible

(b) A positive definite symmetric matrix.

Sum 7.1.5

This is invertible.

Symmetric positive definite is if and only if all λs are positive. Thus, matrix does not have 0, as an E.V, and is invertible, since there are no zeroes on the diagonals, $\det \neq 0$.

(c) A matrix A which is similar to an orthogonal matrix B.

Def 3.4.5: A is sim. to B if

Thm 7.3.5: $AS = SB$, or $B = S^{-1}AS$

Since B is orthogonal it is invertible. $\det(B) = \det(A)$ due to similarity therefore $\det(A) \neq 0$, and thus is invertible.

(d) A matrix such that $A\bar{v} = A\bar{w}$ for some vectors $\bar{v} \neq \bar{w}$.

$$\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ or } v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Av_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad Av_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$Av_1 = Av_2$

not invertible

$\det \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = 0$

Invertible matrix properties

7.1.5 ii)

$A\vec{x} = b$ has a unique solution $\vec{x}$ for all $\vec{b}$ in $\mathbb{R}^n$.

Therefore, if $A\vec{v} = A\vec{w}$ for $\vec{v}$ and $\vec{w}$, then this property is not satisfied, and A is not invertible.

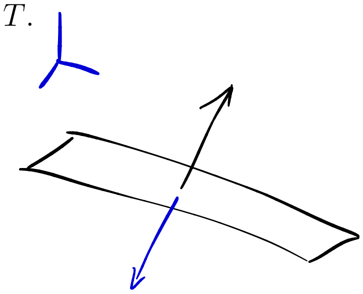
OVER →

6. (9pts) Suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a reflection through the plane P described by the equation

$$-x + 2y + z = 0$$

.

(a) Arguing geometrically, find the eigenvalues for the transformation T.



1 Basis of an eigenspace must be perpendicular, other 2 must be parallel.

$\lambda = 1$ almu 2. The parallels will not change

$\lambda = -1$ almu 1. however the orthogonal will invert

(b) Find a basis for the each eigenspace of T.

normal $= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$\frac{1}{a^2+b^2+c^2} \begin{bmatrix} -a^2+b^2+c & -2ab & -2ac \\ -2ab & a^2-b^2+c^2 & -2bc \\ -2ac & -2bc & a^2+b^2-c^2 \end{bmatrix}$

$\begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix} = T$

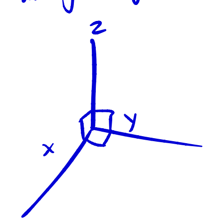
Since $\lambda = -1$, it is associated w/ the perpendicular vector (orthogonal to reflection) Since the normal is $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, this is the basis for λ_{-1}.

As with planes, the other two will be orthogonal to the reflected basis i.e

$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \lambda_1$

normalized λ_1 basis: $\begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix}$

all right angles



(c) Suppose that A is the matrix such that $T(\bar{x}) = A\bar{x}$. Explain why how you know that this matrix must be symmetric, even without calculating it.

For a reflection A, $A^{-1} = A^T$ because it reverts the reflection. A must be symmetric for $A^T = A$, we must be able to undo the reflection to its starting point, and thus, it must be symmetric.

Columns are orthogonal unit vectors that are linearly independent. $(A^TA = A^{-1}A)$

A is symmetric if each eigenvector from an eigenvalue is orthogonal to an eigenvector of a different value

$\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} +1 \\ 0 \\ 1 \end{pmatrix} = 0 \quad -1 + 1 = 0$

$\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} = 0 \quad 2 - 2 = 0$

$\Big\}$ orthogonal

OVER →

7. (8pts) There is a population of owls and squirrels living together in a forest. Let O_t be the population of owls at time t, and S_t be the population of squirrels at time t in thousands. Because the owls prey on the squirrels, both populations are dependent on the other. This can be modeled by the discrete dynamical system such that

$$\begin{bmatrix} O_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.3 \\ -0.4 & 1.3 \end{bmatrix} \begin{bmatrix} O_t \\ S_t \end{bmatrix}$$

This coefficient matrix has eigenvalue 1.1 with eigenvector $\bar{v}_1 = (1, 2)$ and eigenvalue 0.7 with eigenvector $\bar{v}_2 = (3, 2)$.

Use this information to answer the following questions.

(a) Suppose that $\bar{x}_0$ is a vector representing the initial number of owls and squirrels and write $\bar{x}_0 = a\bar{v}_1 + b\bar{v}_2$. Compute $A^t\bar{x}_0$ in terms of a, b and t.

$x_0 = a(1,2) + b(3,2)$ $\quad A^t x_0$

$A = 1.1^t x_0 \Rightarrow \quad 1.1^t \left(a\begin{pmatrix}1\\2\end{pmatrix}\right) + .7^t \left(b\begin{pmatrix}3\\2\end{pmatrix}\right)$

$A^t = SBS^{-1} \quad A^t x_0 = SBS^{-1}\begin{pmatrix}a\\b\end{pmatrix}$

$S\begin{bmatrix}1.1^t & 0\\ 0 & 0.7^t\end{bmatrix}S^{-1}\begin{bmatrix}a\\b\end{bmatrix}$

$\begin{bmatrix}1.1^t & 3\cdot 0.7^t\\ 2\cdot 1.1^t & 2\cdot 0.7^t\end{bmatrix}\begin{bmatrix}-.5 & .75\\ .5 & -.25\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}$

$A^t x_0 \quad 1.1^t \cdot a\begin{bmatrix}1\\2\end{bmatrix} + .7^t \cdot b\begin{bmatrix}3\\2\end{bmatrix}$

* UPE NOTE, I MISSED 1pt FOR THIS

(b) Compute $lim_{t\to\infty}A^t\bar{x}_0$ in terms of a and b.

$\lim_{t\to\infty} A^t x_0$ $\quad$ $O(t)$: $a\underbrace{(1.1)^t}_{\infty} + 3b\underbrace{(.7)^t}_{0}$ $\Big\} a\infty$

$S(t)$: $2a\underbrace{(1.1)^t}_{\infty} + 2b\underbrace{(.7^t)}_{0}$ $\Big\} 2a\infty$

as $t\to\infty$ term $\to 0$.

$\therefore \quad \lim_{t\to\infty} 2a\infty \Rightarrow \lim_{t\to\infty} a\infty$ ↳ this accounts for signs

ratio: $\frac{a\infty}{2a\infty} = \frac{1}{2}$ (or 2)

$= \begin{bmatrix}-.5\cdot 1.1^t + \overbrace{1.5\cdot .7^t}^{0} & .75\cdot 1.1^t - \overbrace{.75\cdot .7^t}^{0}\\ -1.1^t + \underbrace{.7^t}_{0} & 1.5\cdot 1.1^t - \underbrace{.5\cdot .7^t}_{0}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} \xRightarrow{t\to\infty}$

$\overset{t\to\infty}{=} 1.1^t\begin{bmatrix}-\frac{1}{2} & +\frac{3}{4}\\ -1 & +\frac{3}{2}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = 1.1^\infty \Rightarrow \infty$

(c) Use your answer from part c to describe the long term dynamics of this system. For what values of a and b will the owl and squirrel populations decline and eventually the animals will go extinct, and for what values of a and b will the populations continue to grow?

$= \begin{bmatrix}-.5\cdot 1.1^t + \overbrace{1.5\cdot .7^t}^{0} & .75\cdot 1.1^t - \overbrace{.75\cdot .7^t}^{0}\\ -1.1^t + \underbrace{.7^t}_{0} & 1.5\cdot 1.1^t - \underbrace{.5\cdot .7^t}_{0}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} \xRightarrow{t\to\infty}$

$\overset{t\to\infty}{=} 1.1^t\begin{bmatrix}-\frac{1}{2} & +\frac{3}{4}\\ -1 & +\frac{3}{2}\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = 1.1^\infty \Rightarrow \infty$

If $\frac{1}{2}a > \frac{3}{4}b$, they will trend to extinction

$2a > 3b$

If $3b > 2a$, the population will continue grow

(d) Given that we start with an initial population where the animals do not go extinct, what will be the ratio between the number of owls and the number of squirrels in the long term?

as $t \to \infty$, the ratio approaches 1:2 (1 owl to 2000 squirrels)

$\dfrac{a\overset{\infty}{\cancel{(1.1)}^t} + 3b\overset{0}{\cancel{(.7)}^t}}{2a\overset{\infty}{\cancel{(1.1)}^t} + 2b\overset{0}{\cancel{(.7)}^t}}$

$\dfrac{a + 0\cdot 3b}{2a + 0\cdot 2b} \Rightarrow \dfrac{\cancel{a}}{2\cancel{a}} = \dfrac{1}{2}$