

Interpret the transformation first as a pure rotation, then as a pure scaling. Give the parameters in both cases.

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As a pure rotation, we can see that since $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $\cos\theta = -1$ and $\sin\theta = 0$,
meaning θ is π . \vec{x} is rotated 180° , effectively flipped in terms of direction.
This is $T(\vec{x}) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \vec{x}$ for $\theta = \pi$ radians. ✓

As a pure scaling, if \vec{x} is a matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $T(\vec{x})$ will be transformed into
 $\begin{pmatrix} -a & -b \\ -c & -d \end{pmatrix}$, not stretching or compressing but pointing in the opposite
direction of \vec{x} . This is $T(\vec{x}) = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \vec{x}$ for $a = -1$. ✓

$$(0 \ 1 \ -1)$$

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We find $\text{rref}(A)$.

$$\text{rref}(A) = \begin{pmatrix} 2 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \div 2 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} - R_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix} + 2R_2$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \times -1 = \text{rref}(A) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

ROWSPAN: all nonzero rows of $\text{rref}(A)$: $\underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}}_{\text{rowspan}}$ ✓

COLUMNSPAN: columns in rref with leading 1's but in original matrix: $\underbrace{\begin{pmatrix} 2 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{columnspan}}$ ✓