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$2n$ $\binom{i}{k-i}$ x x x x x x / x x x x x x

Question 1

Use a counting argument to show that $2^{2n} = \sum_{k=0}^{2n} \sum_{i=\max(0,k-n)}^{k} \binom{n}{i}\binom{n}{k-i}$.

L.H. 2^{2n} = Counts ~~the cardinality of~~ all possible subsets of a set with $2n$ elements

R.H. $\binom{n}{i}$ denotes the number of ways you may count subset of $|i|$ from n elements

$\binom{n}{k-i}$ denotes the number of ways you may count subsets of $|k-i|$ from n elements.

$\binom{n}{i}\binom{n}{k-i}$ denotes the number of ways you may create two subsets, one with i cardinality, one with $k-i$ cardinality, from n elements → creating two subsets whose total cardinality is k from the same n elements

$\sum_{k=0}^{2n} \sum_{i=\max(0,k-n)}^{k} \binom{n}{i}\binom{n}{k-i}$ is the same as picking all possible ways you may pick i elements (from $0 < i < k-n$), then choosing from the same elements, choosing ($k-i$ elements) from a set of n, which effectively creates all possible subsets of $2n$ cardinality. Therefore $2^{2n} = \sum_{k=0}^{2n} \sum_{i=\max(0,k-n)}^{k}$

from the same then k won't be k

This is the same as choosing two subsets, whose sum is from 0 to $2n$, from a set of n elements

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Question 2

[a] A bin of light bulbs has six bad light bulbs and twelve good light bulbs. We pick seven lightbulbs uniformly at random from this bin. Let A be the event that at most two of them are defective. What is $P(A)$?

that doesn't match description

x x x x x x
o o o o o 7
o o o o o 7

$A_1\ A_2\ A_3\ A_4\ A_5\ A_6$

$P(A) = 1 - P(\bar{A})$

$$P(A) = 1 - \frac{\binom{12}{3}\binom{6}{3} + \binom{12}{2}\binom{6}{4} + \binom{12}{1}\binom{6}{5} + \binom{12}{0}\binom{6}{6}}{\binom{18}{7}}$$

P(1 we do not pick

[b] Let B be the even that we pick all six defective bulbs. What is $P(A|\overline{B})$? (You may write this in terms of $P(A)$, without writing that out again, whether or not you found it in the previous step.)

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$$P(A|\overline{B}) = \frac{P(A\cap\overline{B})}{P(\overline{B})} = \boxed{\frac{P(A) - \frac{1}{\binom{14}{7}}}{1 - \frac{1}{\binom{14}{7}}}}$$

$$P(B) = \frac{1}{\binom{14}{7}} \qquad P(\overline{B}) = 1 - \frac{1}{\binom{18}{7}}$$

Question 3

$a_2 = 6a_1 - 8a_0$

Solve the following recurrence relation: $a_n = 6a_{n-1} - 8a_{n-2}$, with initial conditions $a_0 = 1$ and $a_1 = 1$.

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$r^2 = 6r - 8$

$r^2 - 6r + 8 = 0$

$(r-4)(r-2) = 0$

$r_1 = 4 \quad r_2 = 2.$

$a4^n + b2^n = ?$

I.C. $\Rightarrow a4^0 + b2^0 = 1$

$a + b = 1$

$a4^1 + b2^1 = 1$

$\begin{pmatrix} 4a + 2b = 1 \\ a + b = 1 \end{pmatrix}$

$4a + 2b = 1$

$-\ 2a + 2b = 2$

$2a = -1$

$a = -\frac{1}{2}$

$b = \frac{3}{2}$

$$\boxed{a_n = -\frac{1}{2}\cdot 4^n + \frac{3}{2}\cdot 2^n}$$

$a_2 = \frac{3}{2}4$

5	6	10