33A - Midterm 1

Name:

UID:

Question 1

(a) *[3 points]* Define the span of a set of vectors.

(b) *[4 points]* Explain whether or not the vector $\vec{x} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ is in the span of the vectors $\vec{y} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$ and $\vec{z} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$.

(c) *[3 points]* Suppose that $\vec{v}_1$ is in the span of $\vec{v}_2, \vec{v}_3$. Does this imply that $\vec{v}_3$ is in the span of $\vec{v}_1, \vec{v}_2$?

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Question 2

(a) *[3 points]* Explain geometrically what it means to project a vector in 2D onto a line L.

(b) *[4 points]* Find the projection of the vector $\vec{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ onto the line $L = span\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$.

(c) *[3 points]* Use the projected vector you found in (b) to find the reflection of $\vec{x}$ in the line L. (An incorrect answer from (b) will not cost points here.)

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Question 3

(a) *[2 points]* Define the kernel of a matrix.

(b) *[3 points]* Show that for any $m \times n$ matrix A, saying that the $\ker(A) = \{\vec{0}\}$ is the same as saying that the columns of A are independent.

(c) *[3 points]* Show that if $\vec{x}, \vec{y} \in \ker(A)$, and $\vec{z} \in \text{span}(\vec{x}, \vec{y})$, then $\vec{z} \in \ker(A)$.

(d) *[2 points]* Do elementary row operations preserve the kernel of a matrix?

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	Score
1	
2	
3	
Σ	