Problem 1. *Determine whether the vector $\vec{x}$ is in the span of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$, and if so, find the coefficients,*

$$\vec{x} = \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix}; \quad \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 1 \\ 1 & 3 & 6 & 3 \end{array}\right] \xrightarrow[R_3 - R_1 \to R_3]{R_2 - R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & -6 \\ 0 & 2 & 5 & -4 \end{array}\right] \xrightarrow[R_3 - 2R_2 \to R_3]{R_1 - R_2 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 13 \\ 0 & 1 & 2 & -6 \\ 0 & 0 & 1 & 8 \end{array}\right]$$

$$\xrightarrow[R_2 - 2R_3 \to R_2]{R_1 + R_3 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & -22 \\ 0 & 0 & 1 & 8 \end{array}\right].$$

Good.

$$\vec{x} = 21\vec{v}_1 - 22\vec{v}_2 + 8\vec{v}_3$$

$$\begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix} = 21\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 22\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 8\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$$

$7 = 21 - 22 + 8$

$= -1 + 8 = 7$ ✓

$1 = 21 - 44 + 24$

$= -23 + 24 = 1$ ✓

$3 = 21 - 66 + 48$

$= -45 + 48 = 3$ ✓