MATH 33A EXAM 2

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Name: Minh Le

SID: 9044 78767

Name of TA: Bon-Soon

Discussion section: D

I have read and understood the Student Honor Code, and this exam reflects my unwavering commitment to the principles of academic integrity and honesty expressed therein.

Signature:

Each part of each problem is worth the number of points stated in parentheses. You must show all work to get any partial credit, which will be awarded for certain progress in a problem only if no substantially false statements have been written.

Please write your answers to problems A-E in the box below.

Problem	Answer	Points
1A	number	1
1B	subspace	1
1C	true	0
1D	5	2
1E	π/4	2
1F	AB	2

8

This side for instructor's use only:

Problem	Points
2a	6
2b	0
3a	6
3b	7
3c	2
4	3

27

Problem 1. For each question below, you are given a set of possible answers. Select the single answer which best answers the question, and write it on the blank provided. You must also write your answer on the front page of the exam in the box provided.

There is no partial credit on this problem.

number (A) (1 point) The rank of an $m \times n$ matrix is what type of thing?
{ number, vector, subspace, matrix, linear transformation }

subspace (B) (1 point) The span of the columns of an $m \times n$ matrix is what type of thing?
{ number, vector, subspace, matrix, linear transformation }

True (C) (2 points) Suppose A is an $n \times n$ matrix such that $\langle \vec{x}, \vec{y} \rangle = 0$ implies $\langle A\vec{x}, A\vec{y} \rangle = 0$. Then A is an orthogonal matrix.
{ True, False }

5 (D) (2 points) Let A be a $\overset{n \times m}{6 \times 7}$ matrix whose image is two dimensional. What is the dimension of $\operatorname{im}(A^T)^\perp$?
$\{0, 1, 2, 3, 4, 5, 6, 7\}$

$\frac{1}{9} + \frac{4}{9} + \frac{4}{9}$

$\pi/4$ (E) (2 points) What is the angle between the vectors $\overset{\vec{v}}{\begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}}$ and $\overset{\vec{w}}{\begin{pmatrix} \sqrt{2} \\ \sqrt{2} \\ 0 \end{pmatrix}}$?

$\left\{\pi, \frac{5\pi}{6}, \frac{3\pi}{4}, \frac{2\pi}{3}, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}, 0\right\}$. $\vec{v} \cdot \vec{w} = \frac{1\sqrt{2}}{3} + \frac{2\sqrt{2}}{3} + 0 = \sqrt{2} = \|\vec{v}\| \|\vec{w}\| \cos\theta$

(F) (2 points) Suppose A and B are $n \times n$ symmetric invertible matrices. Which of the following is not necessarily symmetric?
$\{\cancel{A^T}, A^2, \cancel{A^{-1}}, A + B, A - B^T, AB\}$

$\sqrt{2} = (1)(2) \cos$

$\sqrt{2}$ 1 1

64 49 51

36 44

Problem 2. (10 points) You must show all work to get partial credit. You must simplify your work to receive full credit. 49

98 49

4 49

6 (a) (8 points) Compute the QR factorization of the matrix $M = \begin{bmatrix} 1 & 0 & 1 \\ 7 & 7 & 8 \\ 1 & 2 & 1 \\ 7 & 7 & 6 \end{bmatrix}$ $\begin{matrix} \\ -7R_1 \\ -R_1 \\ -7R_1 \end{matrix}$

$M = QR$ Q is set of o.n. vects

Q's column vect are O.N.B

$\mathrm{proj}\ \frac{\vec v \cdot \vec w}{\vec w \cdot \vec w} \vec w$

$\vec u_1 = \begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix}$ norm'l'n

$\vec u_2 = \begin{pmatrix} 0 \\ 7 \\ 2 \\ 7 \end{pmatrix} - \mathrm{proj}_{u_1} \vec v_2 = \begin{pmatrix} 0 \\ 7 \\ 2 \\ 7 \end{pmatrix} - \frac{100}{100}\begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$\vec u_3 = \begin{pmatrix} 1 \\ 8 \\ 1 \\ 6 \end{pmatrix} - \frac{\begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 8 \\ 1 \\ 6 \end{pmatrix}}{10} \begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix} - \frac{(\ )(\ )}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$Q = \begin{bmatrix} 1/10 & -1/\sqrt{2} & 0 \\ 7/10 & 0 & 1/\sqrt{2} \\ 1/10 & 1/\sqrt{2} & 0 \\ 7/10 & 0 & -1/\sqrt{2} \end{bmatrix}$ 4x3 3x3

$= \begin{pmatrix} 1 \\ 8 \\ 1 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 7 \\ 1 \\ 7 \end{pmatrix} - 0$

$= \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$

$r_{11} = \|\vec v_1\|$

$r_{jj} = \|\vec v_j\|$

$r_{ij} = \vec u_i \cdot \vec v_j$ for $i<j$

$\frac{49}{10}\ \frac{2}{10}\ \frac{49}{10}$

(b) (2 points) Compute the matrix of the projection onto $V = \mathrm{im}(M)$.

$\mathrm{proj}_V \vec x = \vec x_{\parallel}$

$r = \begin{bmatrix} 10 & 10 & 10 \\ 0 & \cancel{10} & 2 \\ 0 & 0 & \cancel{10\sqrt{2}} \end{bmatrix}$

6

Problem 3. (10 points) You must show all work to get partial credit. You must simplify your work to receive full credit.

Consider $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$.

3×2 2×1

(a) (6 points) Find a vector $\vec{x}^*$ which minimizes $\|A\vec{x}^* - \vec{b}\|$.

$\vec{x}^* = (A^TA)^{-1}A^T\vec{b}$

$A^T = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}_{2\times 2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\vec{b}$

$A^TA = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\vec{x}^* = \frac{1}{3}\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$ 3

$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$(A^TA)^{-1} = \left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$

$= \frac{1}{3}\begin{bmatrix} 6 \\ 6 \end{bmatrix}$ typecked

$\left[\begin{array}{cc|cc} 0 & -3 & 1 & -2 \\ 1 & 2 & 0 & 1 \end{array}\right] \div 3, \ +\frac{2}{3}r_1$

$\boxed{\vec{x}^* = \begin{bmatrix} 2 \\ 2 \end{bmatrix}}$

$\left[\begin{array}{cc|cc} 0 & 1 & -1/3 & 2/3 \\ 1 & 0 & \frac{2}{3} & -1/3 \end{array}\right]$

$1 - \frac{4}{3}$

(b) (2 points) Verify that $A\vec{x}^* - \vec{b}$ is orthogonal to im(A).

$A\vec{x}^* - \vec{b}$

if $\vec{X}$ is orthogonal to im A, it means for $\vec{v}$ in V where V is the basis for im(A), $\vec{X}\cdot\vec{v} = 0$

$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$ ✓

$\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0$ ✓

(c) (2 points) Compute the matrix of the projection onto im(A).

$A\underbrace{(A^TA)^{-1}}A^T = P$

$\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

$\boxed{P = \frac{1}{3}\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}}$

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Problem 4. (10 points) Let A be an $m \times n$ matrix and $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \in \mathbb{R}^n$. Suppose that $A\vec{v}_1, A\vec{v}_2, A\vec{v}_3, A\vec{v}_4$ are linearly independent. Show that $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ are linearly independent.

- let i, j be 2 different # betwn 1 & 4 (so $\vec{V_i} \neq \vec{V_j}$)
- A is $m \times n$ matrix with column vectors $\vec{x}_1, \vec{x}_2 \ldots \vec{x}_n$)

$$A = \begin{bmatrix} | & | & & \\ \vec{x}_1 & x_2 & \ldots & x_n \\ | & | & & \end{bmatrix}$$

- $A\vec{V_1}, A\vec{V_2} \ldots A\vec{V_4}$ are linearly independent if there is a trivial relation between each vector

$$\begin{bmatrix} | & | & & | \\ V_1 & V_2 & \ldots & V_n \\ | & | & & | \end{bmatrix}$$

if $C_1 A\vec{V}_1 + C_2 AV_2 \cdots C_4 A_4 = \vec{0}$

then $\begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix} = \vec{0}$

$$V^t \begin{bmatrix} - V_1 - \\ - V_2 - \\ - \end{bmatrix}$$

$$\vec{0} = C_1(\vec{x}_1 V_{1,1} + \vec{x}_2 V_{1,2} \cdots \vec{x}_n(V_{1,n})) + \ldots C_n(\vec{x}_1 V_{n,1} + \vec{x}_2 V_{n,2} \cdots \vec{x}_n V_{n,n})$$

$$\vec{0} = (C_1 V_{1,1} + C_2 V_{2,1} \ldots C_n V_{n,1})\vec{x}_1 \ldots + (C_1 V_{1,n} \cdots C_n V_{n,n})\vec{x}_n$$

Define matrix

B as $\begin{bmatrix} | & & | \\ V_1 & \ldots & V_n \\ | & & | \end{bmatrix}$

$$B^T = \begin{bmatrix} - V_1 - \\ - V_2 - \\ \vdots \\ - V_n - \end{bmatrix}$$

a column of that is

$$\begin{pmatrix} V_{1,1} \\ V_{2,1} \\ V_{3,1} \\ \vdots \\ V_{n,1} \end{pmatrix} \cdots$$

$\vec{0}$

linear independence rem

$$C_1\vec{V}_1 \ldots C_n\vec{V}_n = \vec{0} \qquad \begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix} = \vec{0}$$