MATH 33A EXAM 1

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I have read and understood the Student Honor Code, and this exam reflects my unwavering commitment to the principles of academic integrity and honesty expressed therein.

Signature:

Each part of each problem is worth the number of points stated in parentheses. You must show all work to get any partial credit, which will be awarded for certain progress in a problem only if no substantially false statements have been written.

**Please write your answers to problems A-E in the box below.**

| Problem | Answer | Points |
|---|---|---|
| 1A | FALSE | 2 |
| 1B | FALSE | 2 |
| 1C | FALSE | 2 |
| 1D | FALSE | 2 |
| 1E | TRUE | 2 |

10

**This side for instructor's use only:**

| Problem | Points |
|---|---|
| 2a | 5 |
| 2b | 5 |
| 3a | 5 |
| 3b | 3 |
| 3c | 2 |
| 4 | 7 |

27

37

**Problem 1.** Below is a list of statements. Decide which are true and which are false. On the left of each, write "TRUE" or "FALSE" in capital letters. You must also write your answer ("TRUE" or "FALSE" in capital letters) on the front page of the exam.

There is no partial credit on this problem.

FALSE (A) (2 points) Suppose $A$ is $m \times n$. The vector $A\vec{x}$ for $\vec{x} \in \mathbb{R}^n$ is a linear combination of the rows of $A$.

colums

FALSE (B) (2 points) Every square matrix is invertible.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

one to one, square matrix can be 1 to 1

FALSE (C) (2 points) Suppose $A$ is $m \times n$, and $A\vec{x} = \vec{b}$ has a unique solution for some vector $\vec{b} \in \mathbb{R}^m$. Then $n > m$.

$n = m$

not zero vector?

FALSE (D) (2 points) Suppose $A$ is $m \times n$, with $n > m$. Then $\ker(A) = \{0\}$.

TRUE (E) (2 points) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one, then $\ker(T) = \{\vec{0}\}$.

unique so only

$\begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = \vec{0}$

$n = m$

STOP

one to one so only 1 vector maps to $\vec{0}$.

**Problem 2.** (*10 points*) You must show all work to get partial credit.

(a) (5 points) Use Gaussian elimination to compute the inverse of $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$.

5

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right] -3r_1$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right] +2R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & -1 & -3 & 1 \end{array}\right] \times -1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-15 \qquad 6 \quad -5$

$$\boxed{A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) (5 points) Find a $2 \times 3$ matrix $A$ and a $3 \times 2$ matrix $B$ such that $AB = I_2$, but $BA \neq I_3$.
*Hint: You can do this only using 0's and 1's for the entries of A and B.*

5 0

A B

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

<u>focus on pivots</u>

$$\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$ag + bi + ck = 1$ this will take too long.

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ heh.

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = B$$

checkers

**Problem 3.** (*10 points*) You must show all work to get partial credit.

Consider the $4 \times 5$ matrix $A = \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ✓

(a) (5 points) Find a set of vectors in $\mathbb{R}^5$ which spans $\ker(A)$.

$$\left(\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \begin{array}{l} a = -2s - 3t \\ b = s \\ c = -2t \\ d = -t \\ e = t \end{array} = \left[\begin{array}{c|c} -2 & -3 \\ 1 & 0 \\ 0 & -2 \\ 0 & -1 \\ 0 & 1 \end{array}\right]$$

free free

$s, t$ are free variable $\in \mathbb{R}$

2 free variable
2 independent vectors needed

$\ker(A)$ is span of column vectors matrix representing solution equations

$$= \boxed{\operatorname{span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \\ 1 \end{bmatrix}\right)} \quad 2$$

✓

checkers

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \checkmark$$

(b) (3 points) Find a set of vectors in $\mathbb{R}^4$ which spans $\operatorname{im}(A)$.

span of im is just column vectors of A

$$\operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}\right)$$

not final answer

if you look carefully though, not all these vectors are linearly ind. while the def. of span does not require so, we will still simplify.

$$\boxed{\operatorname{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}\right)} \quad 3$$

✓

or $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ ...

(c) (2 points) If you answered both part (a) and part (b) correctly, you get the final 2 points for this question if the number of vectors you found in part (a) plus the number of vectors you found in part (b) equals 5.

$2 + 3 = 5$ ✓

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**Problem 4.** *(10 points)* Suppose we have $j$ vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j \in \mathbb{R}^n$ and $k$ vectors $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_k \in \operatorname{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j\}$. Prove that any linear combination of $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_k$ is also a linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j$.

*Note: You are being asked to prove $V = \operatorname{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j\}$ is closed under taking linear combinations. You may not assume $V$ is a subspace in this question, since that would beg the question!*

. Let $\vec{x}$ be a linear combination of set of $\vec{w}$ vectors in ~~the~~ span $\{\vec{v}_1, \vec{v}_2, \vec{v}_3 \ldots \vec{v}_j\}$

$$\vec{x} = \sum_{a=1}^{k} c_a \vec{w}_a \quad \text{when } c_a \in \mathbb{R}$$

. $\vec{w}$ however is a vector of the span of $(\vec{v}_1, \vec{v}_2 \ldots \vec{v}_k)$ By definition

$\vec{w} = \sum_{b=1}^{j} d_b \vec{v}_b$ where $d \in \mathbb{R}$ ($\vec{w}$ is a linear combination of any vector in that set)

scalar multiple for linear combination of $j$ vectors

therefore .. $\vec{x} = \sum_{a=1}^{k} c_a \sum_{b=1}^{j} d_b \vec{v}_b$

which means $\vec{x}$ is just a linear combination of a linear combination of the $j$ vectors $(\vec{v}_1 \ldots \vec{v}_k)$ so $\vec{x}$ is just a linear combination of $j$ vectors $(\vec{v}_1 \ldots \vec{v}_k)$

$\blacksquare$

= linear combination of a linear combination