MATH 33A EXAM 1 Name: _

October 19, 2015 SID: _

Name of TA: _

I have read and understood the Student Honor Code, and this exam reflects my unwavering commitment to the principles of academic integrity and honesty expressed therein.

Signature:___ ________________________

Each part of each problem is worth the number of points stated in parentheses. You must show all work to get any partial credit, which will be awarded for certain progress in a problem only if no substantially false statements have been written.

Please write your answers to problems A-E in the box below

Problem	Answer	Points
1A	FALSE	2
1B	TRUE	0
1C	FALSE	2
1D	TRUE	2
1E	FALSE	2

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This side for instructor's use only:

Problem	Points
2a	5
2b	5
3a	5
3b	3
3c	2
4	10

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Problem 1. Below is a list of statements. Decide which are true and which are false. On the left of each, write "TRUE" or "FALSE" in capital letters. You must also write your answer ("TRUE" or "FALSE" in capital letters) on the front page of the exam.

There is no partial credit on this problem.

FALSE (A) (2 points) Suppose A is $m \times n$, with $\ker(A) = \{0\}$. Then $m = n$.

TRUE (B) (2 points) If $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto, then $\ker(T) = \{\vec{0}\}$.

FALSE (C) (2 points) Every square matrix is invertible.

TRUE (D) (2 points) Suppose A is $m \times n$, and $A\vec{x} = \vec{b}$ has a unique solution for some vector $\vec{b} \in \mathbb{R}^m$. Then $n \leq m$.

FALSE (E) (2 points) Suppose A is $m \times n$. The vector $A\vec{x}$ for $\vec{x} \in \mathbb{R}^n$ is a linear combination of the rows of A.

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \\ 8 \end{bmatrix}$$

$\mathbb{R}^3$

$a + b\,3c = 4$

$2a + b + 6c = 8$

Problem 2. (*10 points*) You must show all work to get partial credit.

(a) (5 points) Use Gaussian elimination to compute the inverse of $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$.

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$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & -1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$$\boxed{A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}}$$

(b) (5 points) Find a 2×3 matrix A and a 3×2 matrix B such that $AB = I_2$, but $BA \neq I_3$.
Hint: You can do this only using 0's and 1's for the entries of A and B.

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$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\boxed{A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$BA = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq I_3$$

Problem 3. (*10 points*) You must show all work to get partial credit.

Consider the 4×5 matrix $A = \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. already RREF

(a) (5 points) Find a set of vectors in $\mathbb{R}^5$ which spans $\ker(A)$.

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

x_2 and x_5 free vars

$x_1 + 2x_2 + 3x_5 = 0 \to x_1 = -2x_2 - 3x_5$

$x_3 + 2x_5 = 0 \to x_3 = -2x_5$

$x_4 + x_5 = 0 \to x_4 = -x_5$

let $x_2 = s$, $x_5 = t$

$$\vec{x} = \begin{bmatrix} -2s-3t \\ s \\ -2t \\ -t \\ t \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \\ 1 \end{bmatrix} t$$

$$\boxed{\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \\ 1 \end{bmatrix} \right\}}$$

$\ker(A) = \operatorname{span}\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -2 \\ -1 \\ 1 \end{bmatrix} \right)$

(b) (3 points) Find a set of vectors in $\mathbb{R}^4$ which spans $\operatorname{im}(A)$.

~~RRE~~ A is already RREF

columns w/ leading 1 in RREF(A) that correspond to A

$$\boxed{\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}}$$

$\operatorname{im}(A) = \operatorname{span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)$

(c) (2 points) If you answered both part (a) and part (b) correctly, you get the final 2 points for this question if the number of vectors you found in part (a) plus the number of vectors you found in part (b) equals 5.

$3 + 2 = 5$ ✓

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Problem 4. (*10 points*) Suppose we have j vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j \in \mathbb{R}^n$ and k vectors $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_k \in \text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j\}$. Prove that any linear combination of $\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_k$ is also a linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j$.

Note: You are being asked to prove $V = \text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j\}$ is closed under taking linear combinations. You may not assume V is a subspace in this question, since that would beg the question!

let $\vec{y} \in \text{span}\{\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_k\}$

then $\vec{y}$ can be written as a linear combination of $\vec{w}_1, \ldots, \vec{w}_k$

$$\vec{y} = c_1\vec{w}_1 + c_2\vec{w}_2 + \ldots + c_k\vec{w}_k$$

Remember, $\vec{w}_1, \ldots, \vec{w}_k \in \text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_j\}$, meaning $\vec{w}_1, \ldots, \vec{w}_k$ can be written as a lin. combination of $\vec{v}_1, \ldots, \vec{v}_j$.

So,

*Note: $d_{11}, \ldots, d_{1j}, d_{21}, \ldots d_{kj} \in \mathbb{R}$

$$\vec{y} = c_1(d_{11}\vec{v}_1 + d_{12}\vec{v}_2 + \ldots + d_{1j}\vec{v}_j) + \ldots + c_k(d_{k1}\vec{v}_1 + \ldots + d_{kj}\vec{v}_j)$$

$$= \underbrace{(c_1d_{11} + c_2d_{21} + \ldots + c_kd_{k1})\vec{v}_1 + (c_1d_{12} + \ldots + c_kd_{k2})\vec{v}_2 + \ldots + (c_1d_{1j} + \ldots + c_kd_{kj})\vec{v}_j}_{\text{constant coefficients}}$$

$$\vec{y} = A_1\vec{v}_1 + A_2\vec{v}_2 + \ldots + A_j\vec{v}_j \quad \Big| \quad A_1, \ldots, A_j \in \mathbb{R}$$

$\therefore$ any linear combination of $\vec{w}_1, \ldots, \vec{w}_k$ can be written as a linear combination of $\vec{v}_1, \ldots, \vec{v}_j$