Midterm 1
Linear Algebra and Applications
(Math 33A-001)

Answer the questions in the space provided. If you run out of room for an answer, continue on the back of the page. **Show your work.**

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TA's Name: Sam Yih TA Meeting Day: Thursday

Question:	1	2	3	4	Total
Points:	5	5	5	5	20
Score:	5	5	4	2	16

1. $\boxed{\text{5 points}}$ Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix}, \quad T\begin{pmatrix}-1\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\2\\-6\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}-1\\-1\\2\end{pmatrix} = \begin{pmatrix}-1\\0\\4\end{pmatrix}.$$

Compute the matrix of T.

$T(x) = A\vec{x} \rightarrow$

$A\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$ $\quad A\begin{pmatrix}-1\\1\\1\end{pmatrix} = \begin{pmatrix}3\\2\\6\end{pmatrix}$ $\quad A\begin{pmatrix}-1\\-1\\2\end{pmatrix} = \begin{pmatrix}-1\\0\\4\end{pmatrix}$

$\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{bmatrix}1\\-1\\0\end{bmatrix}$

$\boxed{\begin{array}{l} a = 1\\ d = -1\\ g = 0\end{array}}$

$b = 4 + c$ $\quad -1-4-c+2c=-1$
$c = 4$
$b = 8$

$\Downarrow$

$\begin{bmatrix}1 & b & c\\ -1 & e & f\\ 0 & h & i\end{bmatrix}\begin{pmatrix}-1\\1\\-1\end{pmatrix} = \begin{pmatrix}3\\2\\-6\end{pmatrix}$ $\quad \begin{bmatrix}1 & b & c\\ -1 & e & f\\ 0 & h & i\end{bmatrix}\begin{pmatrix}-1\\-1\\2\end{pmatrix} = \begin{pmatrix}-1\\0\\4\end{pmatrix}$

$-1 + b - c = 3$
$1 + e - f = 2$ if $e = 1+f$
$h - i = -6$
if $h = i - 6$

$-1 - b + 2c = -1$
$1 - e + 2f = 0$
$-h + 2i = 4$
$-i + 6 + 2i = 4$

$\boxed{\begin{array}{l} i = -2\\ h = -8\end{array}}$

$1 - 1 - f + 2f = 0$

$\boxed{\begin{array}{l} f = 0\\ e = 1\end{array}}$

$$\boxed{T = A = \begin{bmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -8 & -2\end{bmatrix}} \qquad \boxed{T(x) = \begin{bmatrix}1 & 6 & 2\\ -1 & 2 & 1\\ 0 & -8 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}}$$

check

$\begin{bmatrix}1 & 8 & 4\\ -1 & 3 & 2\\ 0 & -8 & -2\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\-1\\0\end{bmatrix}$ ✓

$\begin{bmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -8 & -2\end{bmatrix}\begin{bmatrix}-1\\1\\-1\end{bmatrix} = \begin{bmatrix}-1+8-4\\ 1+1+0\\ 0-8+2\end{bmatrix} = \begin{bmatrix}3\\2\\-6\end{bmatrix}$

$\begin{bmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -8 & -2\end{bmatrix}\begin{bmatrix}-1\\-1\\2\end{bmatrix} = \begin{bmatrix}-1-8+8\\ 1-1+0\\ 0+8-4\end{bmatrix} = \begin{bmatrix}-1\\0\\4\end{bmatrix}$

2. $\boxed{\text{5 points}}$ Find a 3×3 non zero matrix A such that $A\vec{x}$ is parallel to the vector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ for all $\vec{x} \in \mathbb{R}^3$.

$\vec{w}$

$x^{\parallel} = \text{proj}_L x$

$\vec{u} = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$(x \cdot u) \cdot u$

$$\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \right) \begin{bmatrix} \frac{1}{\sqrt{3}} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$\left(\frac{1}{\sqrt{3}} x_1 - \frac{1}{\sqrt{3}} x_2 + \frac{1}{\sqrt{3}} x_3 \right) \cdot \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3}x_1 - \frac{1}{3}x_2 + 1/3\, x_3 \\ -1/3\, x_1 + 1/3\, x_2 - 1/3\, x_3 \\ 1/3\, x_1 - 1/3\, x_2 \,.\, 1/3\, x_3 \end{bmatrix}$$

3×3 row col 3×1

1 2 3 12

$$\begin{bmatrix} 2 & 3 & 4 \\ -2 & -3 & -4 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} 2x_1 + 3x_2 + 4x_3 \\ -2x_1 - 3x_2 - 4x_3 \\ 2x_1 + 3x_2 + 4x_3 \end{matrix}$$

So $A = \begin{bmatrix} 2 & 3 & 4 \\ -2 & -3 & -4 \\ 2 & 3 & 4 \end{bmatrix}$

so component / colums are scalars of vector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

3. $\boxed{\text{5 points}}$ Let A be a 4×4 matrix, $\vec{\mathbf{b}}$ is a non-zero vector in $\mathbb{R}^4$, and $\vec{\mathbf{0}}$ is the zero vector in $\mathbb{R}^4$. We are told that the linear system $A\vec{\mathbf{x}} = \vec{\mathbf{0}}$ has ***infinitely*** many solutions. What can you say about the number of solutions of the system $A\vec{\mathbf{x}} = \vec{\mathbf{b}}$? You must explain your answer.

$$A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \vec{0}$$ – is kernel

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if there is infinite solutions then rank < 4.

Then $A\vec{x} = \vec{b}$ has either infinite solutions as well or no solutions.

If there exists infinite solution for $A\vec{x} = \vec{0}$, then there are atleast 2 rows that are identical such that atleast one row is equal to $0\ 0\ 0\ 0 : 0$. This means that one of the components of vector $\vec{x}$ can be anything & it would work. The other components may depend on it.

If $A\vec{x} = \vec{b}$ then there are 2 possible outcomes. It can similarly be infinite if the 2 rows are again identical including the components of $\vec{b}$ at those places, or no solution if the component of $\vec{b}$ are different such that one row would look something like $0\ 0\ 0\ 0 : k$ where k is some #.

4. $\boxed{\text{5 points}}$ Let A be a $m \times n$ matrix and B a $n \times p$ matrix. If $\ker(A) = \{\vec{0}\}$ and $\ker(B) = \{\vec{0}\}$, then prove that $\ker AB = \{\vec{0}\}$.

if $\ker(A) = \{0\}$ & $\ker(B) = \{0\}$

$A\vec{x} = \vec{0}$ B

$$AB\vec{x} = \vec{0}$$
$$\Downarrow$$
$$AB\vec{x} = A\vec{0}$$
$$\Downarrow$$
$$(AB)\vec{x} = \vec{0}$$

so
$\ker AB \supseteq \{\vec{0}\}$ too
since $\vec{x}$ is in AB as well as A & B

and if $\ker(A) = \{0\}$ & $\ker(B) =$
and $(AB)\vec{x} = \vec{0}$
then $\ker(AB) = \{\vec{0}$

have only shown $\{\vec{0}\} \subseteq \ker AB$.