MATH 33A Final

RYAN DALY

TOTAL POINTS

95 / 100

QUESTION 1

1 Problem 1 10 / 10

✓ **+ 5 pts** Correct basis for orthogonal complement of L

✓ **+ 5 pts** Correctly uses Gram Schmidt on the basis obtained from the previous point, it doesn't matter whether it's the right basis

  - 2 pts small mistake in computing orthonormal vectors

  + 0 pts Incorrect basis for orthogonal complement of L

  + 0 pts Incorrectly uses Gram-Schmidt (on basis, obtained from previous point, it doesn't matter whether it's the right basis)

  + 0 pts Missing answer

  + 0 pts Does not use Gram-Schmidt on the basis (correct or not) obtained from the previous point, but on a collection of vectors that form a basis of a different subspace

QUESTION 2

2 Problem 2 8 / 10

✓ **+ 5 pts** Correct basis for the image

✓ **+ 5 pts** Correct basis for the kernel (based on obtained RREF, it doesn't matter if it is the correct RREF)

  + 3 pts Derives the basis for the kernel from the RREF making a small mistake

  + 0 pts Wrong basis for the kernel

✓ **- 2 pts** Small mistake in computing RREF

  + 0 pts Wrong basis for the image

  + 0 pts Missing basis for the kernel

  + 0 pts Missing basis for the image

QUESTION 3

3 Problem 3 10 / 10

✓ **+ 6 pts** Main Solution: Correctly set up system of equations. Includes setting up transition matrix solution.

✓ **+ 4 pts** Main Solution: Correctly solve system.

  - 1 pts Minor arithmetic errors.

  - 2 pts Transition matrix solution incorrect. Did not invert.

  + 0 pts Incorrect.

  + 4 pts +4: Set up wrong system, but solved.

QUESTION 4

4 Problem 4 7 / 10

✓ **+ 5 pts** Reasonable explanation of why det(A^-1)=1/det(A) using interpretation as expansion factor.

  + 3 pts Reasonable explanation why A not invertible implies det(A)=0 via expansion factor interpretation.

✓ **+ 2 pts** Reasonable explanation why det(A)=0 implies A not invertible using expansion factor interpretation.

  + 2 pts Bonus Points: Explained what the expansion factor is.

  + 2 pts Bonus Points: Explained why det(AB)=det(A)det(A) using expansion factors.

  + 3 pts Reasonable explanation of the first property, but not using expansion factors.

  + 3 pts Reasonable explanation of second property but did not use expansion factors.

  + 4 pts Incorrect, but exhibited some understanding of determinant as expansion factor interpretation.

  + 0 pts incorrect

QUESTION 5

5 Problem 5 10 / 10

✓ **+ 3 pts** Sets up system of equations correctly

✓ **+ 3 pts** Solves system of equations correctly

✓ **+ 4 pts** **Geometric interpretation; it is a line in R3**

   + 2 pts Partially credit for geometric interpretation

   + 0 pts No credit

6 Problem 6 **12 / 12**

✓ **+ 2 pts** **Explains why the matrix is diagonalizable [spectral thm or other correct argument]**

   + 1 pts Partial credit on 1st part

   + 0 pts Incorrect answer to 1st part

✓ **+ 10 pts** **Full credit for second part**

   + 1 pts Finds characteristic polynomial

   + 1 pts Eigenvalues

   + 2 pts Multiplicities

   + 3 pts Eigenspaces

   + 3 pts S

   + 0 pts No credit

7 Problem 7 **13 / 13**

✓ **+ 13 pts** **Completely Correct**

   + 4 pts Full Credit Criterion 1: Student describes the eigenspace for eigenvalue 1 and gives its geometric multiplicity.

   + 4 pts Full Credit Criterion 2: Student describes the eigenspace for eigenvalue 0 and gives its geometric multiplicity.

   + 2 pts Full Credit Criterion 3: Student shows that A is diagonalizable.

   + 3 pts Full Credit Criterion 4: Student shows $A^k =$ A, or just
$A^k = SB\ S^{-1}$.

   + 0 pts Incorrect/ no progress made

8 Problem 8 **10 / 10**

✓ **+ 10 pts** **Completely Correct**

   + 5 pts Full Credit Criterion 1: Student describes the linear transformation given by the matrix A. They need only indicate what happens to the standard basis, but they could also describe it geometrically.

   + 5 pts Full Credit Criterion 2: The correct inverse matrix is provided. Ideally the student should just undo the linear transformation described above, but they can also use row reduction.

   + 0 pts Completely Incorrect

   + 3 pts Potential Partial Credit: If the student makes a small error in one of the parts (e.g. forgets a negative sign or writes an index incorrectly) they can get partial credit.

Problem 9 15 pts

9.1 **5 / 5**

✓ **+ 2 pts** **Correct Answer**

✓ **+ 3 pts** **Correct counter example.**

   + 0 pts Incorrect

9.2 **5 / 5**

✓ **+ 2 pts** **Correct answer**

✓ **+ 3 pts** **Essentially correct explanation; states that row operations do not change the solution space or that RREF doesn't change it, but this has to be explicitly stated**

   + 2 pts Circular explanation; we use the RREF of A to find the kernel because they have the same kernel, not the other way around

   + 1 pts Example instead of explanation

   + 1 pts Inaccurate explanation

   + 0 pts False / No credit

9.3 **5 / 5**

✓ **+ 5 pts** **Correct, and justification given**

   + 2 pts Correct, but incorrect or no justification given

   + 0 pts Incorrect

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Final (Math 33A, Fall 2019)

Your Name: Ryan Daly

UCLA id: 505 416 119

Date: 12/09/19

The rules: You can answer using a pencil or ink pen. You are allowed to use only this paper, pencil or pen, and the scratch paper provided. You should not hand the scratch paper in. No calculators. No books, no notebooks, no notes, no mobile phones, no web access. You must write your name and UCLA id. You have exactly 180 minutes.

Problem 1	10 points
Problem 2	10 points
Problem 3	10 points
Problem 4	10 points
Problem 5	10 points
Problem 6	12 points
Problem 7	13 points
Problem 8	10 points
Problem 9	15 points
Total	100 points

Good luck!

DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO

Problem 1 (10 points)

Let L be the line in $\mathbb{R}^3$ spanned by the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Find a basis for the orthogonal complement $L^{\perp}$ of L. Then use Gram-Schmidt to obtain an orthonormal basis for $L^{\perp}$. $\checkmark$

Solution:

$\text{let} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -r-t \\ r \\ t \end{bmatrix} = r\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$v_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \text{basis of } L^{\perp} \text{ is } \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$u_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad v_2^{\perp} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$v_2^{\perp} = \begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix}$

$u_2 = \frac{1}{\sqrt{\frac{6}{4}}}\begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{6}}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

$\text{ONB of } L^{\perp} \text{ is } \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \frac{1}{\sqrt{6}}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

Solution:

Problem 2 (10 points)

Let

$$A = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the image of A and a basis for the kernel of A.

Solution:

$$A = \begin{bmatrix} 0 & -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \div -1 \quad \begin{bmatrix} 0 & 1 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} +I$$

$$\begin{bmatrix} 0 & 1 & 1 & -1 & 0 \\ 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{c} \text{swap I} \\ \text{and II} \end{array} \quad \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$ker = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2t \\ -t+r \\ t \\ r \\ s \end{bmatrix} = t\begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

basis of ker(A) is $\begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

basis of im(A) is $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$

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Solution:

Problem 3 (10 points)

Let $\mathcal{B}$ be a basis of $\mathbb{R}^2$. You know that $\left[\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\left[\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$. What is the basis $\mathcal{B}$? $\checkmark$

Hint: if you are unsure on how to proceed, first recall (and write down) how $\mathcal{B}$-coordinates are defined. You should set up a linear system in four variables and four equations.

Solution:

$$\text{for basis} \quad \overset{v_1}{\begin{pmatrix} a \\ b \end{pmatrix}} \overset{v_2}{\begin{pmatrix} c \\ d \end{pmatrix}}$$

$$1 = 1a + 1c \qquad\qquad -1 = 2a$$
$$2 = 1b + 1d \qquad\qquad 1 = 2b$$

$$1 = -\tfrac{1}{2} + c \quad c = \tfrac{3}{2} \qquad\qquad a = -\tfrac{1}{2} \qquad b = \tfrac{1}{2}$$

$$2 = \tfrac{1}{2} + d \quad d = \tfrac{3}{2} \qquad v_1 = \begin{bmatrix} -\tfrac{1}{2} \\ \tfrac{1}{2} \end{bmatrix} \qquad v_2 = \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix}$$

$$\text{basis } \mathcal{B} \text{ is} \begin{pmatrix} -\tfrac{1}{2} \\ 1/2 \end{pmatrix} \begin{pmatrix} 3/2 \\ 3/2 \end{pmatrix}$$

check

$$\begin{bmatrix} 1 & 0 & 1 & 0 & | & 1 \\ 0 & 1 & 0 & 1 & | & 2 \\ 2 & 0 & 0 & 0 & | & -1 \\ 0 & 2 & 0 & 0 & | & 1 \end{bmatrix} \begin{matrix} \\ \\ -2I \\ -2II \end{matrix} \qquad \begin{bmatrix} 1 & 0 & 1 & 0 & | & 1 \\ 0 & 1 & 0 & 1 & | & 2 \\ 0 & 0 & -2 & 0 & | & -3 \\ 0 & 0 & 0 & -2 & | & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & | & 1 \\ 0 & 1 & 0 & 1 & | & 2 \\ 0 & 0 & 1 & 0 & | & 3/2 \\ 0 & 0 & 0 & 1 & | & 3/2 \end{bmatrix} \begin{matrix} -III \\ -IV \end{matrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 & | & -1/2 \\ 0 & 1 & 0 & 0 & | & 1/2 \\ 0 & 0 & 1 & 0 & | & 3/2 \\ 0 & 0 & 0 & 1 & | & 3/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ 3/2 \\ 3/2 \end{bmatrix}$$
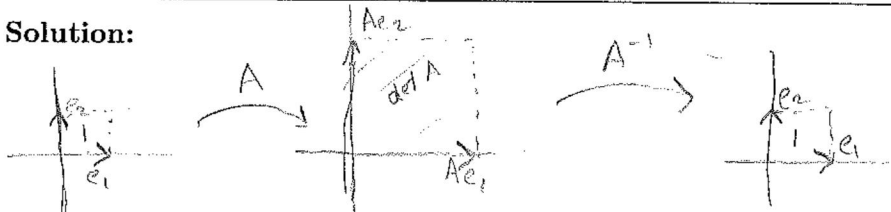
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Solution:

Problem 4 (10 points)

Use the interpretation of the determinant as an expansion factor to explain why the following properties hold:

- For any invertible 2×2 matrix A we have: $\det(A^{-1}) = 1/\det(A)$ ✓

- For any 2×2 matrix A we have: $\det(A) = 0 \Leftrightarrow A$ is not invertible

A drawing and/or a brief explanation are sufficient.

Solution:

Since A^{-1} represents the reverse transformation of A and $AA^{-1} = I_n$, if A has an expansion factor of K, then A^{-1} has an expansion factor of $1/K$. So since the determinant is the expansion factor, $\det(A^{-1}) = 1/\det(A)$

for a 2×2 matrix A, if $\det(A) = 0$ that means that the expansion factor of the transformation A is zero meaning that A condenses all vectors down to a single line or point in $\mathbb{R}^2$. Since it condenses any vector in $\mathbb{R}^2$ to a single point or line, for $Av = w$, there is not a unique v for every w so A is not invertible

Solution:

Problem 5 (10 points) Consider the two planes in $\mathbb{R}^3$ defined by the equations

$$x_1 - x_2 + x_3 = 0 \qquad x_1 + x_2 + x_3 = 0$$

and

$$x_1 = -2x_3$$

$$x_2 + x_3 = 0. \qquad x_2 = -x_3$$

Find all points of intersection of these two planes. Then interpret the points of intersection geometrically (a brief description or a drawing are sufficient).

Solution:

$$\begin{bmatrix} 1 & -1 & 1 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{bmatrix} \xrightarrow{+\text{II}} \begin{bmatrix} 1 & 0 & 2 & | & 0 \\ 0 & 1 & 1 & | & 0 \end{bmatrix}$$

for

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t \\ -t \\ t \end{bmatrix} = t \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

the points of intersection are all points in the $\operatorname{span}\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$ or in other words, the line represented by $\operatorname{span}\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$ in $\mathbb{R}^3$

The two planes intersect at a line

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Solution:

Problem 6 (12 points in total)

Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.

- Without computing eigenvalues and eigenvectors: why is A diagonalisable? $\checkmark$
- Find the eigenvalues of A, compute their algebraic and geometric multiplicities and give a matrix S such that $S^{-1}AS$ is diagonal. $\checkmark$

Solution:

A is diagonalisable since
A is symmetric

$\det \begin{bmatrix} -\lambda & 0 & -1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{bmatrix} = 0$
$\quad \begin{vmatrix} -\lambda & 0 & -1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{vmatrix} \begin{matrix} -\lambda & 0 \\ 0 & -\lambda \\ -1 & 0 \end{matrix}$
$-\lambda^3 + \lambda = 0$

$\lambda(\lambda^2 - 1) = 0 \quad \lambda(\lambda-1)(\lambda+1) = 0$

$\lambda_1 = 0 \quad \lambda_2 = -1 \quad \lambda_3 = 1$

$E_0 = \ker[A - 0] = \ker \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \overset{\text{swap}}{\underset{1\text{and}3}{\to}} = \ker \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix} = \text{span}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
$\qquad v_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$E_{-1} = \ker[A + I_n] \overset{\to}{=} \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \to = \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} r \\ 0 \\ r \end{bmatrix} = \text{span}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
$\qquad v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$E_1 = \ker[A - I_n] = \ker \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \to = \ker \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -r \\ 0 \\ r \end{bmatrix} = \text{span}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$
$\qquad v_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$S = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$
$\qquad$ almu(1) = gemu(1) = 1
$\qquad$ almu(-1) = gemu(-1) = 1
$\qquad$ almu(0) = gemu(0) = 1

Solution:

Problem 7 (13 points) Let V be a plane in $\mathbb{R}^4$, and let A be the matrix that represents the orthogonal projection onto V.

- Is A diagonalisable? If yes, use geometric arguments to find its eigenvalues, and their geometric multiplicities. ✓

- Without computing A: what is A^k, where k is any positive integer? ✓

Solution:

$\lambda = 1$ for all vectors in V since their orthogonal projection onto V is the original vector. There are 2 basis vectors to make up a plane so gemu(1) = dim(V) = 2

$\lambda = 0$ for all vectors that are in $V^\perp$ since their projections onto V will be 0

gemu(0) = dim($V^\perp$) = n − dim(V) = 4−2 = 2

gemu(1) + gemu(0) = 2+2 = 4 = n   so   A is diagonalisable ✓

−A^k is A since $A^k v$ would represent performing the A transformation k times on v. After performing the A transformation once, Av is the orthogonal projection of v onto V. since Av lies in the plane of V, $A(Av) = Av$. Therefore $A^k(v) = A^{k-1}(Av) = Av$

so $A^k = A$

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Solution:

Problem 8 (10 points)

Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

- Describe what the linear transformation represented by this matrix does (it is enough if you describe what the standard basis vectors in $\mathbb{R}^5$ are sent to).

- Compute the inverse of A. **Hint:** there is a simple way to find the inverse using the geometric description from the previous point, without having to use Gauss-Jordan.

Solution:

$T(e_1) = e_5 \quad T(e_2) = e_3 \quad T(e_3) = e_4$

$T(e_4) = e_2 \quad T(e_5) = -e_1$

A^{-1} sends all the transformed basis vectors back to what they originally were

so $A^{-1}(e_1) = -e_5 \quad A^{-1}(e_2) = e_4 \quad A^{-1}(e_3) = e_2$

$A^{-1}(e_4) = e_3 \quad A^{-1}(e_5) = e_1$

$A^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$

Solution:

Problem 9 (15 points total; 5 points each)

Let A be any $n \times n$ matrix. Which of the following are true? Give a brief explanation, or provide a counterexample. Note that for each question you receive 2 points for the correct answer and 3 points for the explanation or counterexample.

1. $\text{im}(A) = \text{im}(\text{RREF}(A))$.

> **Solution:**
> The $\text{im}(A)$ is not necessarily equal to $\text{im}(\text{RREF}(A))$
>
> ex $A \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix}$ $\text{RREF}(A) = \begin{bmatrix} 1 & 3 \\ 0 & 0 \end{bmatrix}$
>
> $\text{im}(A) = \text{span}\begin{pmatrix} 1 \\ 2 \end{pmatrix} \neq \text{im}(\text{RREF}(A)) = \text{span}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

2. $\text{ker}(A) = \text{ker}(\text{RREF}(A))$.

> **Solution:** True
> $\text{ker}(A) = \text{ker}(\text{RREF}(A))$, since the elementary row operations that we perform on A to get $\text{RREF}(A)$ do not change the solution set of the system, A and $\text{RREF}(A)$ have the same solution set so $\text{ker}(A) = \text{ker}(\text{RREF}(A))$

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3. $\det(A) = \det(\text{RREF}(A))$

Solution:

$\det(A)$ is not necessarily equal to $\det(\text{RREF}(A))$

ex $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\text{RREF}(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\det(A) = 2(2) = 4 \neq \det(\text{RREF}(A)) = 1(1) = 1$

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Use this sheet of paper if you need additional space.