# MATH 33A Final

JONATHAN CHAU

TOTAL POINTS

**90 / 100**

QUESTION 1

## 1 Problem 1 10 / 10

✓ **+ 5 pts** **Correct basis for orthogonal complement of L**

✓ **+ 5 pts** **Correctly uses Gram Schmidt on the basis obtained from the previous point, it doesn't matter whether it's the right basis**

**- 2 pts** small mistake in computing orthonormal vectors

**+ 0 pts** Incorrect basis for orthogonal complement of L

**+ 0 pts** Incorrectly uses Gram-Schmidt (on basis, obtained from previous point, it doesn't matter whether it's the right basis)

**+ 0 pts** Missing answer

**+ 0 pts** Does not use Gram-Schmidt on the basis (correct or not) obtained from the previous point, but on a collection of vectors that form a basis of a different subspace

QUESTION 2

## 2 Problem 2 10 / 10

✓ **+ 5 pts** **Correct basis for the image**

✓ **+ 5 pts** **Correct basis for the kernel (based on obtained RREF, it doesn't matter if it is the correct RREF)**

**+ 3 pts** Derives the basis for the kernel from the RREF making a small mistake

**+ 0 pts** Wrong basis for the kernel

**- 2 pts** Small mistake in computing RREF

**+ 0 pts** Wrong basis for the image

**+ 0 pts** Missing basis for the kernel

**+ 0 pts** Missing basis for the image

QUESTION 3

## 3 Problem 3 8 / 10

✓ **+ 6 pts** **Main Solution: Correctly set up system of equations. Includes setting up transition matrix solution.**

✓ **+ 4 pts** **Main Solution: Correctly solve system.**

**- 1 pts** Minor arithmetic errors.

✓ **- 2 pts** **Transition matrix solution incorrect. Did not invert.**

**+ 0 pts** Incorrect.

**+ 4 pts** +4: Set up wrong system, but solved.

QUESTION 4

## 4 Problem 4 7 / 10

✓ **+ 5 pts** **Reasonable explanation of why det(A^-1)=1/det(A) using interpretation as expansion factor.**

**+ 3 pts** Reasonable explanation why A not invertible implies det(A)=0 via expansion factor interpretation.

✓ **+ 2 pts** **Reasonable explanation why det(A)=0 implies A not invertible using expansion factor interpretation.**

**+ 2 pts** Bonus Points: Explained what the expansion factor is.

**+ 2 pts** Bonus Points: Explained why det(AB)=det(A)det(A) using expansion factors.

**+ 3 pts** Reasonable explanation of the first property, but not using expansion factors.

**+ 3 pts** Reasonable explanation of second property but did not use expansion factors.

**+ 4 pts** Incorrect, but exhibited some understanding of determinant as expansion factor interpretation.

**+ 0 pts** incorrect

QUESTION 5

## 5 Problem 5 10 / 10

✓ **+ 3 pts** **Sets up system of equations correctly**

✓ **+ 3 pts** **Solves system of equations correctly**

✓ + **4 pts** **Geometric interpretation; it is a line in R3**

+ **2 pts** Partially credit for geometric interpretation

+ **0 pts** No credit

QUESTION 6

## 6 Problem 6 12 / 12

✓ + **2 pts** **Explains why the matrix is diagonalizable [spectral thm or other correct argument]**

+ **1 pts** Partial credit on 1st part

+ **0 pts** Incorrect answer to 1st part

✓ + **10 pts** **Full credit for second part**

+ **1 pts** Finds characteristic polynomial

+ **1 pts** Eigenvalues

+ **2 pts** Multiplicities

+ **3 pts** Eigenspaces

+ **3 pts** S

+ **0 pts** No credit

QUESTION 7

## 7 Problem 7 13 / 13

✓ + **13 pts** **Completely Correct**

+ **4 pts** Full Credit Criterion 1: Student describes the eigenspace for eigenvalue 1 and gives its geometric multiplicity.

+ **4 pts** Full Credit Criterion 2: Student describes the eigenspace for eigenvalue 0 and gives its geometric multiplicity.

+ **2 pts** Full Credit Criterion 3: Student shows that A is diagonalizable.

+ **3 pts** Full Credit Criterion 4: Student shows A^k = A, or just
A^k =SB S^{-1}.

+ **0 pts** Incorrect/ no progress made

QUESTION 8

## 8 Problem 8 5 / 10

+ **10 pts** Completely Correct

+ **5 pts** Full Credit Criterion 1: Student describes the linear transformation given by the matrix A. They need only indicate what happens to the standard basis, but they could also describe it geometrically.

✓ + **5 pts** **Full Credit Criterion 2: The correct inverse matrix is provided. Ideally the student should just undo the linear transformation described above, but they can also use row reduction.**

+ **0 pts** Completely Incorrect

+ **3 pts** Potential Partial Credit: If the student makes a small error in one of the parts (e.g. forgets a negative sign or writes an index incorrectly) they can get partial credit.

This is the transformation described by the transpose of A.

QUESTION 9

## Problem 9 15 pts

### 9.1 5 / 5

✓ + **2 pts** **Correct Answer**

✓ + **3 pts** **Correct counter example.**

+ **0 pts** Incorrect

### 9.2 5 / 5

✓ + **2 pts** **Correct answer**

✓ + **3 pts** **Essentially correct explanation; states that row operations do not change the solution space or that RREF doesn't change it, but this has to be explicitly stated**

+ **2 pts** Circular explanation; we use the RREF of A to find the kernel because they have the same kernel, not the other way around

+ **1 pts** Example instead of explanation

+ **1 pts** Inaccurate explanation

+ **0 pts** False / No credit

### 9.3 5 / 5

✓ + **5 pts** **Correct, and justification given**

+ **2 pts** Correct, but incorrect or no justification given

+ **0 pts** Incorrect

152

# Final (Math 33A, Fall 2019)

Your Name: Jonathan Chan

UCLA id: 705 166 732

Date: 12/9/19

**The rules:** You can answer using a pencil or ink pen. You are allowed to use only this paper, pencil or pen, and the scratch paper provided. You should not hand the scratch paper in. No calculators. No books, no notebooks, no notes, no mobile phones, no web access. You must write your name and UCLA id. You have exactly 180 minutes.

| | |
|---|---|
| **Problem 1** | 10 points |
| **Problem 2** | 10 points |
| **Problem 3** | 10 points |
| **Problem 4** | 10 points |
| **Problem 5** | 10 points |
| **Problem 6** | 12 points |
| **Problem 7** | 13 points |
| **Problem 8** | 10 points |
| **Problem 9** | 15 points |
| **Total** | 100 points |

**Good luck!**

DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO

**Problem 1** (10 points)

Let $L$ be the line in $\mathbb{R}^3$ spanned by the vector $\begin{bmatrix}1\\1\\1\end{bmatrix}$. Find a basis for the orthogonal complement $L^\perp$ of $L$. Then use Gram-Schmidt to obtain an orthonormal basis for $L^\perp$.

**Solution:**

$$L : \text{Span}\Big(\underbrace{\begin{bmatrix}1\\1\\1\end{bmatrix}}_{v_1}\Big) \qquad \boxed{L^\perp : \text{span}\Big(\underbrace{\begin{bmatrix}1\\-1\\0\end{bmatrix}}_{v_2}, \underbrace{\begin{bmatrix}1\\1\\-2\end{bmatrix}}_{v_3}\Big)}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -1 & 0\end{vmatrix} = +\hat{i} + \hat{j} - 2\hat{k}$$

$$\|v_1\| = \sqrt{1+1+1} = \sqrt{3}$$

$$u_1 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad u_1 \cdot v_2 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix} = \frac{1}{\sqrt{3}}(1-1) = 0$$

$$v_2^\perp = v_2 \qquad \|v_2\| = \sqrt{1^2+(-1)^2} = \sqrt{2} \qquad u_2 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}$$

$$v_3^\perp = v_3 - (u_1 \cdot v_3)u_1 - (u_2 \cdot v_3)u_2 = v_3$$

$$u_1 \cdot v_3 = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1\\1\\-2\end{bmatrix} = \frac{1}{\sqrt{3}}(1+1-2) = 0 \qquad u_2 \cdot v_3 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}\begin{bmatrix}1\\1\\-2\end{bmatrix} = \frac{1}{\sqrt{2}}(1-1) = 0$$

$$\|v_3^\perp\| = \sqrt{1^2+1^2+(-2)^2} = \sqrt{6}$$

$$u_3 = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\1\\-2\end{bmatrix}$$

$$\text{orth. basis of } L^\perp = \boxed{\text{span}\left(\begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\\0\end{bmatrix}, \begin{bmatrix}\frac{1}{\sqrt{6}}\\\frac{1}{\sqrt{6}}\\\frac{-2}{\sqrt{6}}\end{bmatrix}\right)}$$

**Solution:**

**Problem 2** (10 points)

Let

$$A = \begin{array}{c} \overbrace{\phantom{-1\ \ 1\ \ 0}}^{\text{redundant}} \\ \begin{bmatrix} 0 & -1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \end{array}.$$

Find a basis for the image of $A$ and a basis for the kernel of $A$.

**Solution:**

$$\mathrm{im}(A) = \mathrm{span}\left( \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$V_3 = V_2 + 2V_1 \qquad V_3 - V_2 - 2V_1 = 0 \qquad V_5 = 0$

$V_4 = -V_1 - V_2 \qquad V_4 + V_1 + V_2 = 0$

$$-\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\ker(A) = \mathrm{span}\left( \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

**Solution:**

**Problem 3** (10 points)

Let $\mathcal{B}$ be a basis of $\mathbb{R}^2$. You know that $\left[\begin{bmatrix}1\\2\end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix}1\\1\end{bmatrix}$ and $\left[\begin{bmatrix}-1\\1\end{bmatrix}\right]_{\mathcal{B}} = \begin{bmatrix}2\\0\end{bmatrix}$. What is the basis $\mathcal{B}$?

**Hint:** if you are unsure on how to proceed, first recall (and write down) how $\mathcal{B}$-coordinates are defined. You should set up a linear system in four variables and four equations.

**Solution:**

$V_1 = \begin{bmatrix}1\\2\end{bmatrix}$ $\quad V_2 = \begin{bmatrix}-1\\1\end{bmatrix}$ $\quad [V_1]_B = \begin{bmatrix}1\\1\end{bmatrix}$ $\quad [V_2]_B = \begin{bmatrix}2\\0\end{bmatrix}$

$$V_1 \xrightarrow{S^{-1}} [V_1]_B \qquad S^{-1} = B \uparrow \text{ in this case}$$

$B = \begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $\quad [V_1]_B = \begin{bmatrix}a+2b\\ c+2d\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

$[V_2]_B = \begin{bmatrix}-a+b\\ -c+d\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1\\ 0 & 0 & 1 & 2 & 1\\ -1^{+1} & 1^{+2} & 0 & 0 & 2^{+1}\\ 0 & 0 & -1^{+1} & 1^{+2} & 0^{+1} \end{array}\right] \\ \begin{array}{cccc} a & b & c & d \end{array} \end{array} \begin{array}{l} \text{I} \\ \text{II} \\ \text{III} + \text{I} \\ \text{IV} + \text{II} \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1\\ 0 & 0 & 1 & 2 & 1\\ 0 & 3 & 0 & 0 & 3\\ 0 & 0 & 0 & 3 & 1 \end{array}\right] \begin{array}{l} \\ \\ \div 3 \\ \div 3 \end{array} \rightarrow$$

$\longrightarrow$

Go to next page

**Solution:**

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \xrightarrow{\substack{-2\,\mathrm{III} \\ -2\,\mathrm{IV}}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$a = -1$
$b = 1$
$c = \frac{1}{3}$
$d = \frac{1}{3}$

$a + 2b = -1 + 2 = 1$
$c + 2d = \frac{1}{3} + \frac{2}{3} = 1$
$-a + b = -(-1) + 1 = 2$
$-c + d = -\frac{1}{3} + \frac{1}{3} = 0$

$$B = \begin{bmatrix} -1 & 1 \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

**Problem 4** (10 points)

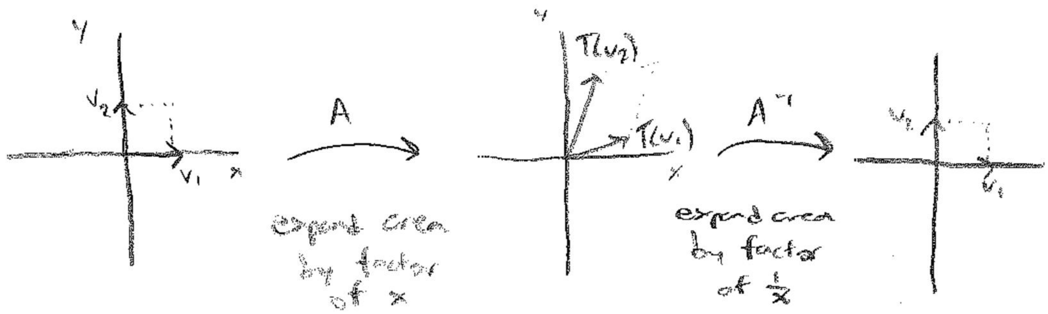
Use the interpretation of the determinant as an expansion factor to explain why the following properties hold:

- For any invertible $2 \times 2$ matrix $A$ we have: $\det(A^{-1}) = 1/\det(A)$
- For any $2 \times 2$ matrix $A$ we have: $\det(A) = 0 \Leftrightarrow A$ is not invertible

A drawing and/or a brief explanation are sufficient.

**Solution:**

Invertible 2×2 matrix



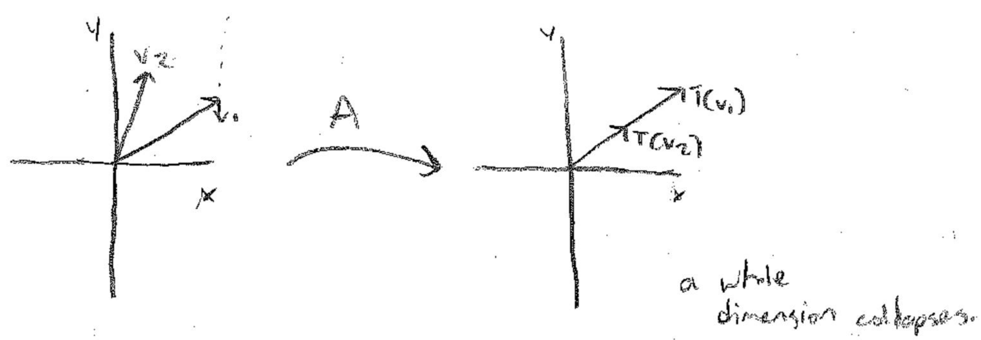

The property that $\det(A^{-1}) = \frac{1}{\det(A)}$ for any invertible matrix $A$ holds because $A^{-1}$ will map the vectors back to before they were transformed by $A$.

So, $\frac{\det(A) \cdot \det(A^{-1})}{\det(A)} = \frac{1}{\det(A)}$

$\det(A^{-1}) = \frac{1}{\det(A)}$ ■

**Solution:**

If $\det(A)=0$, $A$ is not invertible because at least one dimension collapses (as in the resulting transformation will be a line or zerovector), and there is no way to transform a line into an $\mathbb{R}^2$ plane through matrix multiplication since transformations and subspaces are closed under matrix multiplication. ∎




Also if $\det(A)=0$, there's no way to transform an area of 0 back to 1 since $\det(\text{any matrix}\cdot A) = \det(\text{any matrix})\cdot\det(A)=0$.

**Problem 5** (10 points) Consider the two planes in $\mathbb{R}^3$ defined by the equations

$$x_1 - x_2 + x_3 = 0$$

and

$$x_2 + x_3 = 0\,.$$

Find all points of intersection of these two planes. Then interpret the points of intersection geometrically (a brief description or a drawing are sufficient).

**Solution:**

$x_1 - x_2 + \cancel{x_3} = x_2 + \cancel{x_3}$

$x_1 - x_2 = x_2$
$\quad +x_2 \quad +x_2$

$x_1 = 2x_2$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t \\ -t \\ t \end{bmatrix}$$

The points of intersection form a line that spans $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

$$\begin{array}{c} \begin{array}{cccc} & +1 & +1 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} I + II \\ II \end{array} \end{array}$$

$\downarrow$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$x_1 + 2x_3 = 0$

$x_2 + x_3 = 0$

$x_1 = -2x_3$

$x_2 = -x_3$

$x_3 = t$

**Solution:**

**Problem 6** (12 points in total)

Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.

- Without computing eigenvalues and eigenvectors: why is $A$ diagonalisable?
- Find the eigenvalnes of $A$, compute their algebraic and geometric multiplicities and give a matrix $S$ such that $S^{-1}AS$ is diagonal.

**Solution:**

A is diagonalizable because A is symmetric.

$$\begin{bmatrix} -\lambda & 0 & -1 \\ 0 & -\lambda & 0 \\ -1 & 0 & -\lambda \end{bmatrix} \qquad \det(A - \lambda I_n) = (-\lambda)^3 - (-\lambda) = 0$$

$$(-\lambda)((-\lambda)^2 - 1) = 0$$

Eigenvalues: $\lambda = 0 \quad \lambda = 1 \quad \lambda = -1$

$$E_0 = \ker\left(\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\right) = \text{span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) \qquad S = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$E_1 = \ker\left(\begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}\right) = \text{span}\left(\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}\right)$$

$$E_{-1} = \ker\left(\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\right) = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right)$$

almu(0) = 1
almu(1) = 1
almu(−1) = 1
gemu(0) = 1
gemu(1) = 1
gemu(−1) = 1

**Solution:**

$$S^{-1}AS = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$\mathbb{R}^2$

**Problem 7** (13 points) Let $V$ be a plane in $\mathbb{R}^4$, and let $A$ be the matrix that represents the orthogonal projection onto $V$.

- Is $A$ diagonalisable? If yes, use geometric arguments to find its eigenvalues, and their geometric multiplicities.
- Without computing $A$: what is $A^k$, where $k$ is any positive integer?

**Solution:**

$A$ is diagonalizable (because orthogonal projections are diagonalizable)

Eigenvalues: $\boxed{\lambda = 0 \text{ and } \lambda = 1}$

$\boxed{\begin{array}{l} \text{gemu}(0) = 2 \\ \text{gemu}(1) = 2 \end{array}}$

Any vector in $V$ will have an eigenvalue of $1$, and any vector orthogonal to $V$ will have an eigenvalue of $0$.

Since $A$ is diagonalizable, the geometric multiplicties of the eigenvalues must equal their algebraic multiplicities. Since $V$ is a plane, its dimension is 2 and therefore almu$(1) = 2$, meaning gemu$(1) = 2$. Since the domain is $\mathbb{R}^4$, the number of dimensions collapsed into $V$ is 2, meaning almu$(0) = 2$ and gemu$(0) = 2$.

$\boxed{A^k = A}$, since $A^k = (SBS^{-1})^k = SB^kS^{-1} = SBS^{-1} = A$.

$$B^k = \begin{bmatrix} 1^k & 0 & 0 & 0 \\ 0 & 1^k & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = B$$

**Solution:**

**Problem 8** (10 points)

Let $A = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$. $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$

- Describe what the linear transformation represented by this matrix does (it is enough if you describe what the standard basis vectors in $\mathbb{R}^5$ are sent to).
- Compute the inverse of $A$. **Hint:** there is a simple way to find the inverse using the geometric description from the previous point, without having to use Gauss-Jordan.

**Solution:**

$$V_1 \xrightarrow{A} -V_5$$
$$V_2 \xrightarrow{A} V_4$$
$$V_3 \xrightarrow{A} V_2$$
$$V_4 \xrightarrow{A} V_3$$
$$V_5 \xrightarrow{A} V_1$$

It is, in a sense, a reflection.

Since $A$ is an orthogonal matrix, $A^{-1} = A^T$,

so $A^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

**Solution:**

**Problem 9** (15 points total; 5 points each)

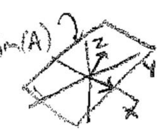
Let $A$ be any $n \times n$ matrix. Which of the following are true? Give a brief explanation, or provide a counterexample. Note that for each question you receive 2 points for the correct answer and 3 points for the explanation or counterexample.

1. $\operatorname{im}(A) = \operatorname{im}(\operatorname{RREF}(A))$.

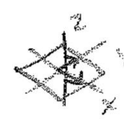
**Solution:**

False, when A is linearly dependent, im(A) spans vectors that may or may not follow the Cartesian axes, while im(RREF(A)) always does.

$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{bmatrix}$ im(A)




im(RREF(A)) =




$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} = \operatorname{rref}(A)$

2. $\ker(A) = \ker(\operatorname{RREF}(A))$.

**Solution:**

True, A and RREF(A) will have the same non-trivial relations, so ker(A) = ker(RREF(A)).

(Doing elementary row operations will not change the non-trivial relations).

3. $\det(A) = \det(\text{RREF}(A))$

**Solution:**

False, When you use Gauss-Jordan Elimination on A to find RREF(A), a row swap will flip the sign of the determinant and a row division by a factor of x will divide the determinant of the matrix by a factor of x.

$\overset{\det = 2}{\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}} = B \quad \overset{\det = 1}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = A = \text{RREF}(B)$ $\det(B) = 2\det(A) = 2\det(\text{RREF}(B))$

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Use this sheet of paper if you need additional space.