Math 33A Exam 1 v1 (Blue/Yellow)

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TOTAL POINTS

50 / 50

QUESTION 1

TF 10 pts

1.1 a **2 / 2**

✓ **- 0 pts** False

1.2 b **2 / 2**

✓ **- 0 pts** True

1.3 c **2 / 2**

✓ **- 0 pts** True

1.4 d **2 / 2**

✓ **- 0 pts** True

1.5 e **2 / 2**

✓ **- 0 pts** False

QUESTION 2

Q2 10 pts

2.1 Given T find A **5 / 5**

✓ **- 0 pts** All Correct

2.2 Given Av1 and Av2 find Av **5 / 5**

✓ **- 0 pts** Correct

QUESTION 3

Inverses 10 pts

3.1 a **5 / 5**

✓ **- 0 pts** Correct

3.2 b **5 / 5**

✓ **- 0 pts** Correct

QUESTION 4

4 Commuting matrices **10 / 10**

✓ **+ 10 pts** Correct

+ 3 pts AX = XA

+ 3 pts Matrix product computation

+ 2 pts Equations from matrices

+ 0 pts Incorrect

QUESTION 5

Matching 10 pts

5.1 scaling **2 / 2**

✓ **- 0 pts** I

5.2 reflection **2 / 2**

✓ **- 0 pts** D

5.3 shear **2 / 2**

✓ **- 0 pts** E

5.4 rotation **2 / 2**

✓ **- 0 pts** C

5.5 projection **2 / 2**

✓ **- 0 pts** G

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2A	Redmond McNamara	T	GEOLOGY 6704
2B		R	ROYCE 154
2C	Albert Zheng	T	ROLFE 3121
2D		R	ROYCE 162
2E	Weiyi Liu	T	BOELTER 5280
2F		R	BOELTER 5436

Section	2	D

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- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
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Page:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

You may use this page for scratch work.

1. (10 points) True/False (circle the correct answer). You do not need to justify your answer.

   Remember that True means **always true**. If a statement is sometimes true, but sometimes false, mark it False.

   (a) If A and B are invertible $n \times n$ matrices then $ABA^{-1}B^{-1} = I_n$.

   True False

   (b) Every vector in $\mathbb{R}^4$ is a linear combination of $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$, and $\vec{e}_4$.

   True False

   (c) If A is a 3×4 matrix of rank 3 then the system $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has infinitely many solutions.

   True False

   $(3\times 4)(4\times 1) = (3\times 1)$

   $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3+x_4 \end{bmatrix}$$

   (d) There exists a nonzero upper triangular 2×2 matrix A such that $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

   True False

   $$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

   (e) The system $A\vec{x} = \vec{b}$ is inconsistent if and only if rref(A) contains a row of zeros.

   True False

2. (10 points) Show your work.

(a) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^4$ is given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_2 - x_3 \\ x_3 \\ 0 \\ \frac{1}{2}x_1 + x_2 \end{bmatrix}.$$

4×3 3×1 4×1

Find a matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x}$ in $\mathbb{R}^3$.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$$

$2x_2 - x_3 = ax_1 + bx_2 + cx_3 \Rightarrow a=0,\ b=2,\ c=-1$

$x_3 = dx_1 + ex_2 + fx_3 \Rightarrow d=0,\ e=0,\ f=1$

$0 = gx_1 + hx_2 + ix_3 \Rightarrow g=h=i=0$

$\frac{1}{2}x_1 + x_2 = jx_1 + kx_2 + lx_3 \Rightarrow j=\frac{1}{2},\ k=1,\ l=0$

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \end{bmatrix}$$

(b) If A is a 4×4 matrix such that

$$A\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad \text{and} \quad A\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

4×4 4×1 4×1

then what is

$$A\begin{bmatrix} 0 \\ 2 \\ -1 \\ 0 \end{bmatrix}?$$

$$A\begin{bmatrix} 0 \\ 2 \\ -1 \\ 0 \end{bmatrix} = A\begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} - A\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix}$$

3. (10 points) For each of the matrices below, either find the inverse or explain why no inverse exists **without using determinants**. Show your work.

(a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{swap rows}} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{array}\right]$$

so the inverse is $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 & 5 \\ 3 & 4 & 2 \\ 1 & 6 & -8 \end{bmatrix} = A$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 5 & 1 & 0 & 0 \\ 3 & 4 & 2 & 0 & 1 & 0 \\ 1 & 6 & -8 & 0 & 0 & 1 \end{array}\right] \xrightarrow[(3)-(1)]{(2)-3(1)} \left[\begin{array}{ccc|ccc} 1 & -1 & 5 & 1 & 0 & 0 \\ 0 & 7 & -13 & -3 & 1 & 0 \\ 0 & 7 & -13 & -1 & 0 & 1 \end{array}\right] \xrightarrow{(3)-(2)} \left[\begin{array}{ccc|ccc} 1 & -1 & 5 & 1 & 0 & 0 \\ 0 & 7 & -13 & -3 & 1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 1 \end{array}\right]$$

no inverse as $\mathrm{rref}(A) \neq I$.

4. (10 points) Find all upper triangular 2×2 matrices of the form

$$X = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$

such that X commutes with **every** 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with a, b, c, d all nonzero. Show your work!

$$AX = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x & y \\ 0 & z \end{bmatrix} = \begin{bmatrix} ax & ay+bz \\ cx & cy+dz \end{bmatrix}$$

$$XA = \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ax+cy & bx+dy \\ cz & dz \end{bmatrix}$$

so $ax = ax + cy$ $\qquad cy = 0$

$cx = cz$ $\qquad cx - cz = 0$

$ay + bz = bx + dy$ $\qquad ay + bz - bx - dy = 0$

$cy + dz = dz$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \\ \downarrow & \downarrow & \downarrow \end{array} \\ \left[\begin{array}{ccc|c} 0 & c & 0 & 0 \\ c & 0 & -c & 0 \\ -b & a-d & b & 0 \end{array}\right] \end{array} \xrightarrow[\text{swap}(1),(2)]{(1)/c,\ (2)/c} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -b & a-d & b & 0 \end{array}\right] \xrightarrow{(3)+b(1)} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & a-d & 0 & 0 \end{array}\right]$$

$$\xrightarrow{(3)-(a-d)(2)} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \text{so} \quad x = z, \ y = 0$$

so $X = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$, $x \in \mathbb{R}$.

5. (10 points) Multiple Choice. Write your answers (A, B, C, etc.) in the boxes provided.

Each of the linear transformations listed below corresponds to one (and only one) of the matrices A through J. Match them up.

$\boxed{\text{I}}$	scaling	$\boxed{\text{D}}$	reflection	$\boxed{\text{E}}$	shear
$\boxed{\text{C}}$	rotation	$\boxed{\text{G}}$	orthogonal projection		

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} -0.6 & 0.8 \\ -0.8 & -0.6 \end{bmatrix} \quad D = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.6 & 0.6 \\ 0.8 & 0.8 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad H = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \quad J = \begin{bmatrix} 0.8 & -0.6 \\ 0.6 & -0.8 \end{bmatrix}$$