Math 33A Final Exam

Jessica (ming Sum Jessica) Cheng

TOTAL POINTS

99 / 100

QUESTION 1

1 Vector in kernel 5 / 5

✓ **- 0 pts** **Correct**

- 5 pts Blank or completely incorrect

- 1 pts Computation mistakes or incorrect form

QUESTION 2

2 Im and Ker of orthogonal projection 5 / 5

✓ **- 0 pts** **Correct**

- 5 pts Blank or completely incorrect

- 1 pts Computation mistakes or wrong dimension

- 1 pts Incorrect form (eg. basis not span)

- 3 pts Show some efforts but not enough (eg. unfamiliar with the definitions or did not find a basis for the plane)

QUESTION 3

Diagonal or rotation-scaling 8 pts

3.1 (a) 4 / 4

✓ **+ 4 pts** **Correct**

+ 2 pts Correct rotation-scaling with real entries based on given complex eigenvalues (or diagonal with complex eigenvalues)

+ 1 pts Rotation-scaling with real entries (or diagonal with complex eigenvalues)

+ 2 pts Eigenvalues 4-i, 4+i

+ 1 pts Used characteristic equation but incorrect or missing eigenvalues

+ 0 pts Incorrect

3.2 (b) 4 / 4

✓ **+ 4 pts** **Correct**

+ 2 pts Correct matrix based on computed eigenvalues

+ 2 pts Eigenvalues 1 and 3

+ 1 pts Used characteristic equation but incorrect or missing eigenvalues

+ 0 pts Incorrect

QUESTION 4

Change of basis 10 pts

4.1 (a) 5 / 6

+ 6 pts Correct

✓ **+ 5 pts** **Minor error (matrix off by one or two entries)**

+ 3 pts Stated B = S^-1 A S

+ 2 pts Computed S^-1

+ 3 pts Stated columns = [A v_i]_B

+ 2 pts B-coordinate vector computation

+ 1 pts Computed AS or S^-1 A

+ 0 pts Incorrect

4.2 (b) 4 / 4

✓ **+ 4 pts** **Correct (diagonal entries of upper triangular B from part (a))**

+ 2 pts Characteristic equation of A or B

+ 0 pts Incorrect

QUESTION 5

Eigenspaces of symmetric matrices are orthogonal 10 pts

5.1 (a) 4 / 4

✓ **- 0 pts** **Correct**

- 4 pts Blank or completely incorrect

- 3 pts Show efforts but not correct (eg. mistakenly took the transpose or unfamiliar with the definition of symmetry matrix)

- 2 pts Incomplete solution (eg. not using A is symmetric)

5.2 (b) **6 / 6**

- ✓ **- 0 pts Correct**
- **- 6 pts** Blank or completely incorrect
- **- 1 pts** Not rigorous for some details (eg. what if one the eigenvalues is 0 or one is the additive inverse of the other one) or missing conclusion
- **- 4 pts** Show efforts but not enough/correct (eg. use spectral theorem directly instead of proving from part a)
- **- 5 pts** Try something but not complete or incorrect (eg. A is not necessarily invertible or orthogonal)

QUESTION 6

Eigenstuff 12 pts

6.1 (a) **3 / 3**

- ✓ **- 0 pts Correct**
- **- 2 pts** Missing eigenvalue
- **- 1 pts** One incorrect multiplicity
- **- 3 pts** Incorrect

6.2 (b) **5 / 5**

- ✓ **- 0 pts Correct**
- **- 2 pts** Incorrect basis for eigenvalue 1
- **- 2 pts** Incorrect basis for eigenvalue 0
- **- 5 pts** Incorrect
- **- 1 pts** Incorrect notation

6.3 (c) **2 / 2**

- ✓ **- 0 pts Correct**
- **- 2 pts** Incorrect, based on part b

6.4 (d) **2 / 2**

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect justification
- **- 2 pts** Incorrect, based on previous parts

QUESTION 7

7 SVD **10 / 10**

- ✓ **- 0 pts Correct**
- **- 8 pts** Serious mistakes; serious conceptual misunderstanding; perhaps U or V isn't orthogonal or the dimensions are wrong.
- **- 3 pts** A computational mistake
- **- 10 pts** Incorrect
- **- 5 pts** Multiple computational mistakes
- **- 7 pts** Wrong dimensions but otherwise close

QUESTION 8

TF 20 pts

8.1 (a) **2 / 2**

- ✓ **- 0 pts True**
- **- 2 pts** False

8.2 (b) **2 / 2**

- **- 2 pts** True
- ✓ **- 0 pts False**

8.3 (c) **2 / 2**

- ✓ **- 0 pts True**
- **- 2 pts** False
- **- 2 pts** No answer

8.4 (d) **2 / 2**

- **- 2 pts** True
- ✓ **- 0 pts False**

8.5 (e) **2 / 2**

- **- 2 pts** True
- ✓ **- 0 pts False**

8.6 (f) **2 / 2**

- ✓ **- 0 pts True**
- **- 2 pts** False

8.7 (g) **2 / 2**

- **- 2 pts** True
- ✓ **- 0 pts False**

8.8 (h) **2 / 2**

- ✓ **- 0 pts True**
- **- 2 pts** False

8.9 (i) **2 / 2**

- **- 2 pts** True
- ✓ **- 0 pts False**

8.10 (j) **2 / 2**

- ✓ **- 0 pts True**
- **- 2 pts** False

QUESTION 9

Fill in the blanks 20 pts

9.1 (a) **2 / 2**

- ✓ **- 0 pts J**
- **- 2 pts** Not J

9.2 (b) **2 / 2**

- ✓ **- 0 pts A**
- **- 2 pts** not A

9.3 (c) **2 / 2**

- ✓ **- 0 pts F**
- **- 2 pts** not F

9.4 (d) **2 / 2**

- ✓ **- 0 pts B**
- **- 2 pts** not B

9.5 (e) **2 / 2**

- ✓ **- 0 pts None**
- **- 2 pts** not None

9.6 (f) **2 / 2**

- ✓ **- 0 pts H**
- **- 2 pts** not H

9.7 (g) **2 / 2**

- ✓ **- 0 pts E**
- **- 2 pts** not E

9.8 (h) **2 / 2**

- ✓ **- 0 pts H and/or K**
- **- 2 pts** neither H nor K

9.9 (i) **2 / 2**

- ✓ **- 0 pts J**
- **- 2 pts** not J

9.10 (j) **2 / 2**

- ✓ **- 0 pts D**
- **- 2 pts** not D

MATH 33A | Final Exam | Spring 2019

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2A	Redmond McNamara	T	GEOLOGY 6704
2B		R	ROYCE 154
2C	Albert Zheng	T	ROLFE 3121
2D		R	ROYCE 162
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2F		R	BOELTER 5436

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Page:	1	2	3	4	5	6	7	8	Total
Points:	10	8	10	10	12	10	20	20	100
Score:									

1. (5 points) Suppose that A is a 5×4 matrix of the form

$$A = \begin{bmatrix} | & | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \\ | & | & | & | \end{bmatrix}.$$

Given that the vector $\begin{bmatrix} 7 \\ 3 \\ -2 \\ 8 \end{bmatrix}$ is in the kernel of A, write $\vec{v}_4$ as a linear combination of the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$. Box your answer.

$$7\vec{v_1} + 3\vec{v_2} - 2\vec{v_3} + 8\vec{v_4} = \vec{0}$$

$$8\vec{v_4} = -7\vec{v_1} - 3\vec{v_2} + 2\vec{v_3}$$

$$\boxed{v_4 = \frac{-7}{8}\vec{v_1} - \frac{3}{8}\vec{v_2} + \frac{1}{4}\vec{v_3}}$$

2. (5 points) Suppose that the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the orthogonal projection onto the plane $3x_1 + x_2 - 2x_3 = 0$. Find a basis for $\text{im}(T)$ and a basis for $\ker(T)$. Box your answers.

$$x_2 = -3x_1 + 2x_3$$

$$= -3s + 2t$$

$$x_1 = s \quad x_3 = t$$

$$\begin{bmatrix} s \\ -3s+2t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} t$$

2) $\boxed{\text{basis of } \text{im}(T) : \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}}$

basis of Ker(T) is normal vector of plane

2) $\boxed{\text{basis of } \ker(T) : \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}}$

3. (8 points) For each of the 2×2 matrices A below, there is an invertible matrix S such that $B = S^{-1}AS$ is either a diagonal matrix $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ or a rotation-scaling matrix $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. Find B in each case (you do not have to find S).

(a) $A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}$

char. eqn.

$$\det(A - \lambda I) = \lambda^2 - \operatorname{tr}A \cdot \lambda + \det A$$

$$0 = \lambda^2 - 8\lambda + 17$$

$$\lambda = \frac{8 \pm \sqrt{64 - 68}}{2} = \frac{8 \pm 2i}{2} = 4 \pm i \qquad \lambda_1 = 4+i, \ \lambda_2 = 4-i$$

$a = 4, \ b = 1$

$$B = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$$

(b) $A = \begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}$

char. eqn.

$$0 = \lambda^2 - 4\lambda + 3$$

$$= (\lambda - 3)(\lambda - 1) \qquad \lambda_1 = 3, \ \lambda_2 = 1$$

$$B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

4. (10 points) Let $T(\vec{x}) = A\vec{x}$ be the linear transformation with matrix

$$A = \begin{bmatrix} 4 & -3 & 2 \\ -2 & 3 & 2 \\ 5 & -5 & 4 \end{bmatrix}.$$

(a) Let $\mathcal{B}$ be the basis of $\mathbb{R}^3$ given by

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \qquad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \vec{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}.$$

Find the matrix of T in the basis $\mathcal{B}$.
(Another way to say this is: find a matrix B such that $[T(\vec{x})]_{\mathcal{B}} = B[\vec{x}]_{\mathcal{B}}$.)

$$\begin{bmatrix} 4 & -3 & 2 \\ -2 & 3 & 2 \\ 5 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{\mathbb{B}}$$

$$\begin{bmatrix} 4 & -3 & 2 \\ -2 & 3 & 2 \\ 5 & -5 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}_{\mathbb{B}}$$

$$\begin{bmatrix} 4 & -3 & 2 \\ -2 & 3 & 2 \\ 5 & -5 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}_{\mathbb{B}}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

(b) Find the eigenvalues of A, repeating any eigenvalues according to their algebraic multiplicities. (So if $\lambda_2 = 17$ has algebraic multiplicity 2, list it twice.)
Hint: use your answer from part (a), which should be nice enough that you don't need to compute a determinant for this part.

Similar matrices have same eigenvalues. A and B are similar.
eigenvalues of triangular matrix are on diagonal

$\Rightarrow$ eigenvalues: $\lambda_1 = 1, \ \lambda_2 = 5, \ \lambda_3 = 6$

5. (10 points) Make sure to fully justify your answers on this page.

Suppose that A is a symmetric $n \times n$ matrix (this is an assumption for both (a) and (b) below).

(a) Show that $A\vec{v} \cdot \vec{w} = \vec{v} \cdot A\vec{w}$ for any two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$.
Hint: remember that another way to write the dot product is $\vec{x} \cdot \vec{y} = \vec{x}^T \vec{y}$.

$$A\vec{v} \cdot \vec{w} = (A\vec{v})^T \vec{w}$$

$$= \vec{v}^T (A^T \vec{w})$$

(A symmetric)

$$= \vec{v}^T A\vec{w}$$

$$= \vec{v} \cdot A\vec{w}$$

(b) Suppose that $\vec{v}$ and $\vec{w}$ are eigenvectors of A with eigenvalues λ and μ, respectively. Show that if $\lambda \neq \mu$ then $\vec{v}$ is orthogonal to $\vec{w}$.

$$A\vec{v} \cdot \vec{w} = \lambda \vec{v} \cdot \vec{w} = \lambda(\vec{v} \cdot \vec{w})$$

$$\vec{v} \cdot A\vec{w} = \vec{v} \cdot \mu \vec{w} = \mu(\vec{v} \cdot \vec{w})$$

$A\vec{v} \cdot \vec{w} = \vec{v} \cdot A\vec{w}$, so $\lambda(\vec{v} \cdot \vec{w}) = \mu(\vec{v} \cdot \vec{w})$.

$\lambda \neq \mu$, so $(\vec{v} \cdot \vec{w})$ must equal to 0.

dot product is 0, therefore $\vec{v}$ orthogonal to $\vec{w}$.

6. (12 points) Define

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues of A and state the algebraic multiplicity of each eigenvalue. (No justification needed for this part.)

0, $\text{almu}(0) = 2$

1, $\text{almu}(1) = 2$.

(b) Find a basis for each eigenspace.

$E_0 = \ker(A) = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)$ basis: of E_0 $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$E_1 = \ker(A - I) = \ker \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{span}\left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right)$ basis of E_1 = $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

(c) State the geometric multiplicity of each eigenvalue.

$\text{gemu}(0) = 2$

$\text{gemu}(1) = 1$

(d) Is A diagonalizable? Justify your answer.

no, because $\sum \text{gemu} = 3 \neq \sum \text{almu} = 4$

$\mathcal{B} = S^{-1}AS \qquad A = SBS^{-1}$

7. (10 points) For this question, it may be helpful to know that

$$\underbrace{\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}}_{S} \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}}_{S^{-1}}.$$

Find a singular value decomposition $A = U\Sigma V^T$ of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Write your answer in the form $\boxed{U = \text{something}, \Sigma = \text{something}, V = \text{something}}$.

$$M = A^TA = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

eigenvalues: $\lambda_1 = 3, \quad \lambda_2 = 2, \quad \lambda_3 = 0$

singular values: $\sigma_1 = \sqrt{3}, \quad \sigma_2 = \sqrt{2}$

$$\vec{v_1} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \quad \vec{v_2} = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \quad \vec{v_3} = \begin{bmatrix} 1/\sqrt{6} \\ -2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

$$\vec{u_1} = \frac{1}{\sqrt{3}} A\vec{v_1} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\vec{u_2} = \frac{1}{\sqrt{2}} A\vec{v_2} = \frac{1}{2} \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \quad V = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

check: $A = U\Sigma V^T$

$$\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{U} \underbrace{\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix}}_{V^T} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{works}$$

8. (20 points) True/False (circle the correct answer). You do not need to justify your answer. Remember that True means **always true**. If a statement is sometimes true, but sometimes false, mark it False.

(a) Every 5×5 matrix has a real eigenvalue.

True False

(b) If A is an $n \times n$ matrix such that $\text{im}(A) = \{\vec{0}\}$ then A is invertible.

True False

(c) If A is symmetric and U is orthogonal then UAU^T is symmetric. $(UAU^T)^T = (U^T)^T A^T U^T$

True False $= UA^TU^T$

$= UAU^T$

(d) If A is a symmetric matrix and λ is one of its eigenvalues, then λ is also one of the singular values of A.

True False $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(e) The matrices $\begin{bmatrix} 8 & 6 & 7 \\ 5 & 3 & 0 \\ 9 & 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 4 & 8 & 1 \\ 5 & 1 & 6 \\ 23 & 4 & 2 \end{bmatrix}$ are similar.

$tr = 12$ $tr = 7$

True False

(f) If $\ker(A^3) = \ker(A^4)$ and $\vec{v}$ is a vector in $\ker(A^5)$, then $\vec{v}$ is automatically in $\ker(A^4)$.

$\vec{x} \in \ker A^5$

$A^5\vec{x} = \vec{0}$

$A^4(A\vec{x}) = \vec{0}$ $A\vec{x} \in \ker A^4 = \ker A^3$

$A^3(A\vec{x}) = \vec{0}$ $A^4\vec{x} = \vec{0}$

True False

(g) If A is a 4×7 matrix then $\text{rank}(A) + \dim(\ker(A)) = 4$.

n m

True False

(h) If A is a symmetric $n \times n$ matrix such that $A^n = 0$ then $A = 0$.

True False

(i) If A is an upper triangular $n \times n$ matrix such that $A^n = 0$ then $A = 0$.

True False $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(j) If A is an $n \times n$ matrix such that $\|A\vec{x}\| = \|\vec{x}\|$ for all $\vec{x}$ in $\mathbb{R}^n$ then A is an orthogonal matrix.

True False

9. (20 points) Fill in the blanks with the matrices below (just write A, B, etc. in each blank). You may use some items more than once, and there are some items that you will not use at all. **If none of the matrices on this list makes the statement true, write NONE.**

λ^2+1

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad F = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad K = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(a) Suppose that $M\vec{x} = \vec{b}$ is a linear system with 3 equations and 3 unknowns. If the system has a unique solution then the reduced row echelon form of M equals __J__.

(b) Suppose that R is a 3×2 matrix and that S is a 2×3 matrix. If $\text{im}(R) = \ker(S)$ then $SR =$ __A__.

$\mathbb{R}^2 \xrightarrow[R]{} \mathbb{R}^3 \xrightarrow[S]{} \mathbb{R}^2$

(c) If M is a 2×2 rotation by $\pi/6$ radians counterclockwise, then $M^{18} =$ __F__.

rotation by 3π ($180°$) $\cos\theta = -1$, $\sin\theta = 0$

(d) $M =$ __B__ is a 2×2 matrix such that $\text{im}(M) = \ker(M)$.

(e) $M =$ __NONE__ is a 3×3 matrix such that $\text{im}(M) = \ker(M)$.

rank-nullity

(f) $\lambda = 0$ is the only eigenvalue of __H__. Furthermore, the algebraic multiplicity of λ is 3 and the geometric multiplicity of λ is 1.

(g) tr(__E__) = 1.

(h) $M =$ __H__ is a matrix with rank 2 whose kernel is not $\{\vec{0}\}$. 2 correct answers

(i) If M is an orthogonal 3×3 matrix then $M^T M = M M^T =$ __J__.

(j) __D__ has complex (non-real) eigenvalues.

You may use this page for scratch work.