- You have 5 minutes   • No calculators   • Show sufficient work

1. Find the reduced row-echelon form of A. Then find a basis of the image of A and of the kernel of A.

$$A = \begin{pmatrix} 1 & 0 & -4 \\ -1 & 2 & 0 \\ 0 & -2 & 4 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 0 & -4 \\ -1 & 2 & 0 \\ 0 & -2 & 4 \end{bmatrix} + I\!I = \begin{bmatrix} 1 & 0 & -4 \\ -1 & 0 & 4 \\ 0 & -2 & 4 \end{bmatrix} + I\!I$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 4 \\ 0 & -2 & 4 \end{bmatrix} \underset{2}{\curvearrowright} = \begin{bmatrix} -1 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & -2 & 4 \end{bmatrix} \cdots -1$$

$$= \begin{bmatrix} 1 & 0 & -4 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \cdots -\tfrac{1}{2} = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} x_3 = t \\ \Rightarrow x_2 = 2t \\ x_1 = 4t \end{array}$$

$$\text{basis}(\text{im}(A)) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix}$$

$$\text{basis}(\ker(A)) = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$