MATH 33A

STUDENT NAME:	
STUDENT ID NUMBER:	
DISCUSSION SECTION NUMBER:	
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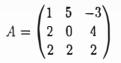
Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work—your work must both justify and clearly identify your final answer. No books, notes or calculators are allowed. You must simplify results of function evaluations when it is possible to do so.

Page	e Points	Score
2	8	8
3	8	8
4	4	4
5	8	8
6	6	4
7	10	10
8	6	6
Total	: 50	48

For instructor use only

1. Consider the matrix



(a) [4 pts] Find a basis for ker(A), which is one-dimensional.

Therefore, one of the basis in Ker(A) is (i)

(b) [2 pts] Using your kernel computation, or otherwise, write down a linear relation between the columns of A that shows that the third column can be viewed as redundant. No further explanation necessary.

The relation is $-2\begin{pmatrix} 1\\2\\2 \end{pmatrix} + \begin{pmatrix} 5\\0\\2 \end{pmatrix} + \begin{pmatrix} -3\\4\\2 \end{pmatrix} = 0$ third column =

(c) [2 pts] Write down a basis for im(A). You may use any of the previous parts of the problem with no further explanation, even if you couldn't solve them. Or you may solve this problem from scratch using row reduction.

 $\begin{pmatrix} 1\\ 2\\ 2 \end{pmatrix}$ and $\begin{pmatrix} 5\\ 0\\ 2 \end{pmatrix}$.

1. State 1.

1

 $\leq V$

(d) [8 pts] Convert your answer from the previous part of the problem into an orthonormal basis for im(A). (Keep your work well-organized!)

(e) [4 pts] Use your work from the previous part of the problem to write a M = QR factorization for a relevant matrix M. (Part of the problem is deciding what M should be!)

$$\begin{bmatrix} 1 & 5 \\ 2 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{25}{5} \\ \frac{1}{3} & \frac{-5}{5} \\ \frac{1}{3} & 0 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 0 & 2\sqrt{5} \end{bmatrix} \checkmark$$

- Feb. 25, 2019
- 2. Suppose we have a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$, and suppose further that we have a subspace V in the range \mathbb{R}^n . We will define a new subset of the *domain* \mathbb{R}^m called the *pre-image* of V as follows:

 $\operatorname{PreIm}(V) := \{ \overrightarrow{\mathbf{x}} \text{ in } \mathbb{R}^m \, | \, T(\overrightarrow{\mathbf{x}}) \text{ is in } V \}.$

That is, $\operatorname{PreIm}(V)$ consists of all vectors in \mathbb{R}^m that get mapped by T to a vector that is in V.

(a) [8 pts] Prove that $\operatorname{PreIm}(V)$ is a subspace of the domain \mathbb{R}^m by checking all three necessary conditions.

To prove
$$Pre Im(V)$$
 is a subspace, we need to check the
following 3 things.
1. $\overline{0}$ is in $Pre Im(V)$.
We have $\overline{0}$ in \mathbb{R}^m that goes to $T(\overline{0}) = \overline{0}$ has
a linear transformation, which $\overline{+}\overline{+}$ is in V asine V is
a subspace. Therefore, $\overline{0}$ is in $Pre Im(V)$.
2. $PreIm(V)$ closed under addition.
If we have $\overline{\pi}_1, \overline{\pi}_2$ in $Pre Im(V)$
then $T(\overline{\pi}_1), T(\overline{\pi}_2)$ is in V .
since V is a subspace and T is linear.
 $T(\overline{\pi}_1 + \overline{\pi}_2) = T(\overline{\pi}_1) + T(\overline{\pi}_2)$ is also in V .
So $\overline{\pi}_1 + \overline{\pi}_2$ is in $Pre Im(V)$, it is closed under addition
if $\overline{\pi}_1$ is in $Pre Im(V)$.
Then $T(\overline{\pi}_2)$ is in V .
Since V is a subspace, and T is linear.
 $T(\overline{\pi}_1 + \overline{\pi}_2) = T(\overline{\pi}_1) + T(\overline{\pi}_2)$ is also in V .
So $\overline{\pi}_1 + \overline{\pi}_2$ is in $Pre Im(V)$.
Then $T(\overline{\pi}_2)$ is in V .
Since V is a subspace, and T is linear.
 $T(K_{\overline{\pi}_1}) = K T(\overline{\pi}_1)$ is also in V .
So $K_{\overline{\pi}_1}$ is in $Pre Im(V)$, $PreIm(V)$ closed under scalar
 $T(K_{\overline{\pi}_2}) = K T(\overline{\pi}_1)$ is also in V .
So $K_{\overline{\pi}_1}$ is in $Pre Im(V)$, $PreIm(V)$ closed under scalar
 $T(K_{\overline{\pi}_2}) = K T(\overline{\pi}_1)$ is also in V .
So $K_{\overline{\pi}_1}$ is in $Pre Im(V)$, $PreIm(V)$ closed under scalar
 $Multiplicant.
 $PreIm(V)$ is a subspace as all Scondition met.$

50

(b) [2 pts] If we had chosen the subspace V to be all of the range \mathbb{R}^n , what subspace would $\operatorname{PreIm}(V)$ be? No explanation necessary.

Prelm(V) will be all of R^m



(c) [2 pts] If we had chosen the subspace V to be the zero subspace $\{\vec{0}\}$ in the range \mathbb{R}^n , what subspace would $\operatorname{PreIm}(V)$ be? No explanation necessary.

Prelm(V) will be € ker(T).

- (d) [2 pts] Circle the one correct statement out of the four choices below (hint: think about $T: \operatorname{PreIm}(V) \to V$ rather than $T: \mathbb{R}^m \to \mathbb{R}^n$).
 - $\dim(\operatorname{PreIm}(V)) = \dim(V)$ regardless of V and/or T.

• $\dim(\operatorname{PreIm}(V)) \ge \dim(V)$ regardless of V and/or T.

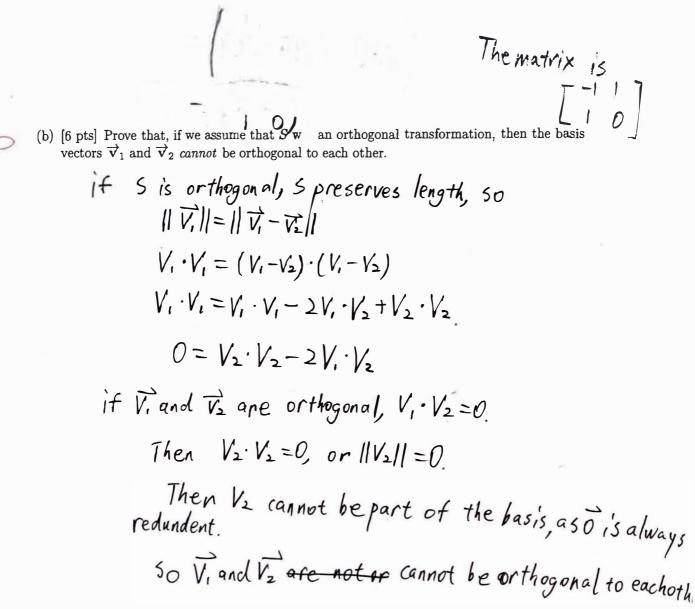
T does not need to swject onto V

- $\dim(\operatorname{PreIm}(V)) \leq \dim(V)$ regardless of V and/or T.
- In some cases $\dim(\operatorname{PreIm}(V)) > \dim(V)$. In other cases $\dim(\operatorname{PreIm}(V)) < \dim(V)$. Finally, there are also cases where $\dim(\operatorname{PreIm}(V)) = \dim(V)$.

3. Let $\mathcal{B} = \{ \overrightarrow{\mathbf{v}}_1, \overrightarrow{\mathbf{v}}_2 \}$ be a basis for \mathbb{R}^2 , and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ be two linear transformations that treat this basis in the following manner:

$$T(\overrightarrow{\mathbf{v}}_1) = \overrightarrow{\mathbf{v}}_2, \quad T(\overrightarrow{\mathbf{v}}_2) = \overrightarrow{\mathbf{v}}_1, \qquad S(\overrightarrow{\mathbf{v}}_1) = \overrightarrow{\mathbf{v}}_1 - \overrightarrow{\mathbf{v}}_2, \quad S(\overrightarrow{\mathbf{v}}_2) = \overrightarrow{\mathbf{v}}_2$$

(a) [4 pts] Write down the matrix B for the composition $T \circ S$ in \mathcal{B} -coordinates.



- 4. Multiple choice and/or true and false (circle one answer only; no justification needed). In all of the problems below, A is an $n \times m$ matrix. $R \xrightarrow{m} R \xrightarrow{n} R$
 - (a) [2 pts] What can we say about $\dim(\operatorname{im}(A)) + \dim(\ker(A))$?

Always $= n$	(Always = m)	Neither of these		
(b) [2 pts] What can we say about $\dim(\ker(A))$? $\bigwedge \mathcal{M}\mathcal{U}[i,t_{\mathcal{Y}}]$ Always $< n$ Always $> n$ Neither of these				
	\sim			

(c) [2 pts] If B is a $p \ge n$ matrix, then we must have $rank(BA) \le rank(A)$.



FALSE

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