5. (a) [2 pts] Write down the 2x2 matrix for rotation by an angle $\theta$.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

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(b) [2 pts] Use the determinant to show that this matrix is invertible.

$$\det(A) = \cos^2\theta + \sin^2\theta = 1$$

$\det(A) \neq 0 \therefore$ The matrix is invertible

(c) [3 pts] Explain geometrically what the inverse matrix should do, and write the inverse matrix down.

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$\therefore$ Inverse $= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ //