

4. Suppose you know that $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$.

(a) [6 pts] Find $(AB)^{-1}$.

$$\begin{aligned} (AB)^{-1} &= B^{-1} \cdot A^{-1} \\ &= \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -2+1 & -1+1 \\ 6+2 & 3+2 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 8 & 5 \end{pmatrix} // \end{aligned}$$

(b) [6 pts] Find B .

$$\begin{aligned} B \cdot B^{-1} &= I_n \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & B &= \begin{pmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{pmatrix} // \\ \begin{pmatrix} -a+3b & a+2b \\ -c+3d & c+2d \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

$$-a+3b=1$$

$$a+2b=0$$

$$-c+3d=0$$

$$c+2d=1$$

$$2b+3b=1$$

$$a=-2b$$

$$c=3d$$

$$3d+2d=1$$

$$b = \frac{1}{5}$$

$$a = -\frac{2}{5}$$

$$c = \frac{3}{5}$$

$$d = \frac{1}{5}$$

(c) [2 pts] What was the rank of A ? (this should require no computations)

square matrix, invertible \Rightarrow rank = 2