

3. [5 pts] Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

Circle all of the following vectors which are members of $\ker(A)$.

$A\vec{x} = \vec{0}$

$\vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \vec{w} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \vec{x} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \vec{y} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \vec{z} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} +(\text{I}) \\ -3(\text{I}) \\ -(\text{I}) \end{matrix} = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} -2(\text{IV}) \\ +2(\text{IV}) \\ \div -1 \end{matrix} = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} -2(\text{II}) \\ \end{matrix} = \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x_1 - x_3 - x_4 = 0$

Declare $x_3 = s, x_4 = t$

$x_2 + x_4 = 0$

$$\Rightarrow \begin{pmatrix} t+s \\ -t \\ s \\ t \end{pmatrix} = t \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

~~$$\begin{pmatrix} 1 & 2 & -1 & 1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix} \begin{matrix} +(\text{I}) \\ -3(\text{I}) \\ -(\text{I}) \end{matrix} = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & -2 \\ 0 & -1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} -2(\text{II}) \\ \end{matrix} = \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$~~

$x_1 - x_3 - x_4 = 0$

$x_1 = t+s$

$x_2 + x_4 = 0$

$x_2 = -s$

t

s

$$= t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$