20S-MATH33A-3 Midterm 1

SHREYA CHATTERJEE

TOTAL POINTS

18 / 25

QUESTION 1

1 4 / 4

✓ **+ 1 pts** **Correct (a) (True)**

✓ **+ 1 pts** **Correct (b) (False)**

✓ **+ 1 pts** **Correct (c) (True)**

✓ **+ 1 pts** **Correct (d) (True)**

QUESTION 2

2 3 / 5

✓ **- 2 pts** **Wrong row operations. Fixing the row operations the conclusions (ii and (iii))would be right, and (i) almost correct.**

Unique solutions if a is different than -4. Infinitely many solutions if a=-4 and b=8. No solutions if a=-4 and b is different than 8.

QUESTION 3

3 5 / 5

✓ **- 0 pts** **(a) correct**

✓ **- 0 pts** **(b) correct**

QUESTION 4

4 6 / 6

✓ **+ 2 pts** **Part 1: The student gets that rank(A) = 2 and null(A) = 2**

✓ **+ 2 pts** **Part 2: The student uses rref(A) to get the first two columns of A are a basis for Im(A).**

✓ **+ 2 pts** **Part 3: The student correctly finds a basis for ker(A).**

+ 0 pts Incorrect

QUESTION 5

5 0 / 5

+ 1 pts (i) Correct formula for projection

+ 1 pts (i) Correct answer (1, 2, 3)

+ 1 pts (ii) Correct answer (-2, -1, 6)

+ 1 pts (iii) Correct method for finding a v (ex. cross product, solving system of equations)

+ 1 pts (iii) Correct solution for v (any non-zero multiple of (-5, 4, -1))

✓ **+ 0 pts** **No points**

Incorrect formula for projection...

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1. a. true b. false
c. true d. true

2.
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 4 & 0 & a & b \end{array}\right] -4\text{I} = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 0 & -8 & a-12 & b-32 \end{array}\right] \div 3 =$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & a-12 & b-32 \end{array}\right] \begin{array}{l} -2\text{II} \\ \\ +8\text{II} \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & a+4 & b-20 \end{array}\right] \div a+4$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & b-20 / a+4 \end{array}\right]$$

i. the system has a unique solution when
a $a \neq -4$ & $b \neq 20$

ii. the system has infinite solutions when $a = -4$ and $b = 20$, making the bottom row [0 0 0] = [0]

iii. the system has no solutions when
$a = -4$ and $b \neq 20$, so the last row
is [0 0 0] = [b], some real number

1 4 / 4

- ✓ **+ 1 pts** Correct (a) (True)
- ✓ **+ 1 pts** Correct (b) (False)
- ✓ **+ 1 pts** Correct (c) (True)
- ✓ **+ 1 pts** Correct (d) (True)

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1. a. true b. false
c. true d. true

2. $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 4 & 0 & a & b \end{array}\right] -4\text{I} = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 3 & 6 & 9 \\ 0 & -8 & a-12 & b-32 \end{array}\right] \div 3 =$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 8 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & a-12 & b-32 \end{array}\right] \begin{array}{l} -2\text{II} \\ \\ +8\text{II} \end{array} = \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & a+4 & b-20 \end{array}\right] \div a+4$

$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & b-20/a+4 \end{array}\right]$

i. the system has a unique solution when
~~a~~ $a \neq -4$ & $b \neq 20$

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2 3 / 5

✓ **- 2 pts Wrong row operations. Fixing the row operations the conclusions (ii and (iii))would be right, and (i) almost correct.**

Unique solutions if a is different than -4. Infinitely many solutions if a=-4 and b=8. No solutions if a=-4 and b is different than 8.

3) a. $A = \begin{bmatrix} 2 & 6 & 12 \\ 0 & 3 & 6 \\ 5 & 0 & 5 \end{bmatrix}$

b. assuming that this matrix does have an inverse:

↳ augment with identity matrix

$$\left[\begin{array}{ccc|ccc} 2 & 6 & 12 & 1 & 0 & 0 \\ 0 & 3 & 6 & 0 & 1 & 0 \\ 5 & 0 & 5 & 0 & 0 & 1 \end{array}\right] \div 2 = \left[\begin{array}{ccc|ccc} 1 & 3 & 6 & 1/2 & 0 & 0 \\ 0 & 3 & 6 & 0 & 1 & 0 \\ 5 & 0 & 5 & 0 & 0 & 1 \end{array}\right] -5\,\mathrm{I}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 3 & 6 & 1/2 & 0 & 0 \\ 0 & 3 & 6 & 0 & 1 & 0 \\ 0 & -15 & -25 & -5/2 & 0 & 1 \end{array}\right] -\mathrm{II} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 0 \\ 0 & 3 & 6 & 0 & 1 & 0 \\ 0 & -15 & -25 & -5/2 & 0 & 1 \end{array}\right] +5\,\mathrm{II}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 0 \\ 0 & 3 & 6 & 0 & 1 & 0 \\ 0 & 0 & 5 & -5/2 & 5 & 1 \end{array}\right] \div 3 = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 0 \\ 0 & 1 & 2 & 0 & 1/3 & 0 \\ 0 & 0 & 5 & -5/2 & 5 & 1 \end{array}\right] \div 5$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 0 \\ 0 & 1 & 2 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -1/2 & 1 & 1/5 \end{array}\right] -2\,\mathrm{III} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 0 \\ 0 & 1 & 0 & 0 & -5/3 & -2/5 \\ 0 & 0 & 1 & -1/2 & 1 & 1/5 \end{array}\right]$$

this matrix is invertible because rank (A) = 3.

the inverse A^{-1} is:

$$\begin{bmatrix} 1/2 & -1 & 0 \\ 1 & -5/3 & -2/5 \\ -1/2 & 1 & 1/5 \end{bmatrix}$$

3 5 / 5

- ✓ **- 0 pts** (a) correct
- ✓ **- 0 pts** (b) correct

4. i. $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 4 & 2 & 2 \\ 0 & 3 & 1 & 3 \end{bmatrix}$

rref:

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 4 & 2 & 2 \\ 0 & 3 & 1 & 3 \end{bmatrix} -2I = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 3 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \div 3$$

$$= \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1/3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} -2I = \begin{bmatrix} 1 & 0 & 1/3 & -1 \\ 0 & 1 & 1/3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank (A) = 2, as there are 2 leading 1s
nullity (A) = 2, as there are 2 free variables

ii. $\text{im}(A) = \left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} \right\}$, as these are the columns pertaining to the leading 1s. these vectors are linearly independent and are a spanning set of A

iii. $x_1 + 0 + \frac{1}{3}x_3 - x_4 = 0$ $\quad x_1 = -\frac{1}{3}x_3 + x_4$
$0 + x_2 + \frac{1}{3}x_3 + x_4 = 0$ $\quad x_2 = -\frac{1}{3}x_3 - x_4$
$x_3 = s$
$x_4 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3}s + t \\ -\frac{1}{3}s - t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} -1/3 \\ -1/3 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\ker(A) = \left\{ \begin{pmatrix} -1/3 \\ -1/3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

4 6 / 6

- ✓ **+ 2 pts** Part 1: The student gets that rank(A) = 2 and null(A) = 2
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- ✓ **+ 2 pts** Part 3: The student correctly finds a basis for ker(A).
- **+ 0 pts** Incorrect

5. ~~Projection = $\vec{x}$ − (x · u) u~~

5b.

$2x + 4y + 6z = 0$

$x = [4, 5, 0]$

$\frac{1}{\|v\|} \vec{v} = \frac{1}{2\sqrt{14}} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ → perpendicular

$\text{ref}(v) = x - 2(x \cdot u)\vec{u} =$

$\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} - \frac{2}{2\sqrt{14}} \left(\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \right) \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$

reflection $= \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{14}} \left[\begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \right]$

a. projection $= x - (x \cdot u) u$

u, from above $= \frac{1}{2\sqrt{14}} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$

$\text{proj} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} - \left(\begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} \cdot \frac{1}{2\sqrt{14}} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} \right) \cdot \frac{1}{2\sqrt{14}} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$

5 0 / 5

+ **1 pts** (i) Correct formula for projection
+ **1 pts** (i) Correct answer (1, 2, 3)
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+ **1 pts** (iii) Correct method for finding a v (ex. cross product, solving system of equations)
+ **1 pts** (iii) Correct solution for v (any non-zero multiple of (-5, 4, -1))

✓ + **0 pts** **No points**

Incorrect formula for projection...