Math 33A/1
Spring 2016
04/22/16
Time Limit: 50 Minutes

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SID Number: [redacted]

Day \ T.A.	David	Casey	Adam
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter your name and SID number on the top of this page, cross the box correspondiing to your discussion section, and put your initials on the top of every page, in case the pages become separated. Also, have your photo ID on the desk in front of you during the exam.

Use a pen to record your answers. Calculators or computers of any kind are not allowed. You are not allowed to consult any other materials of any kind, including books, notes and your neighbors. You may use the back of this sheet for your notes ("scratch paper"). If you need additional paper, let the proctors know.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	10
2	10	9
3	10	10
4	10	10
Total:	40	39

Of course, if you have a question about a particular problem, please raise your hand and one of the proctors will come and talk to you.

J.F

1. (10 points) Consider the following system of linear equations:

$$\left|\begin{aligned} 2x - 4y + \ \ z &= 0 \\ x + ky \qquad &= 0 \\ 2y + kz &= 1 \end{aligned}\right|,$$

where k is a real constant.

(a) For which values of k does the system have a unique solution? No solutions? Infinitely many?

(b) Solve the system when $k = 0$.

if $k=-2$ $\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \to$ unique solution

$$\begin{bmatrix} 2 & -4 & 1 & 0 \\ 1 & k & 0 & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \to \begin{bmatrix} 1 & k & 0 & 0 \\ 2 & -4 & 1 & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} \\ -2(I) \\ \\ \end{matrix} \to \begin{bmatrix} 1 & k & 0 & 0 \\ 0 & -4-2k & 1 & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} \\ /-4-2k \\ \\ \end{matrix} \to \begin{bmatrix} 1 & k & 0 & 0 \\ 0 & 1 & \frac{1}{-4-2k} & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} -k(II) \\ \\ -2(II) \end{matrix}$$

$$\to \begin{bmatrix} 1 & 0 & \frac{k}{4+2k} & 0 \\ 0 & 1 & \frac{1}{-4-2k} & 0 \\ 0 & 0 & k+\frac{2}{4+2k} & 1 \end{bmatrix}$$

$-4-2k \neq 0$

$-2k \neq 4$

$k \neq -2$

$k + 2/(4+2k) \neq 0$

$\frac{4k+2k^2+2}{4+2k} \neq 0$

$4k+2k^2+2 \neq 0$

$k^2+2k+1 \neq 0$

$(k+1)(k+1) \neq 0$

a) System has a unique solution when $k \neq -1$

System has NO solutions when $k=-1$

b) $$\begin{bmatrix} 2 & -4 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & -4 & 1 & 0 \end{bmatrix} \begin{matrix} \\ :2 \\ -2(I) \end{matrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & -4 & 1 & 0 \end{bmatrix} \begin{matrix} \\ \\ +4(II) \end{matrix} \to \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad k \neq -1$$

$x = 0$

$y = 1/2$

$z = 2$

a) To check: IGNORE

$$\begin{bmatrix} 2 & -4 & 1 & 0 \\ 1 & k & 0 & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} :2 \\ \\ \\ \end{matrix} \to \begin{bmatrix} 1 & -2 & \frac{1}{2} & 0 \\ 1 & k & 0 & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} \\ -(I) \\ \\ \end{matrix} \to \begin{bmatrix} 1 & -2 & \frac{1}{2} & 0 \\ 0 & k+2 & -\frac{1}{2} & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} \\ :k+2 \\ \\ \end{matrix} \to \begin{bmatrix} 1 & -2 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2k+4} & 0 \\ 0 & 2 & k & 1 \end{bmatrix} \begin{matrix} +2(II) \\ \\ -2(II) \end{matrix} \to \begin{bmatrix} 1 & 0 & \frac{1}{2}-\frac{1}{k+2} & 0 \\ 0 & 1 & -\frac{1}{2k+2} & 0 \\ 0 & 0 & k+\frac{1}{k+2} & 1 \end{bmatrix} \xrightarrow{\text{if } k=-1} \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \to \text{inconsistent}$$

$k=-2$ $\begin{bmatrix} 1 & -2 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 2 & -2 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 & \frac{1}{2} & 0 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & -\frac{1}{2} & 0 \end{bmatrix}$ ✓

$\frac{k^2+2k+1}{k+2}$

$k^2+2k+1 \neq 0$

$k \neq 1$

2. (10 points) (a) Find the inverse of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix}$.

(b) Find a 4×3 matrix A satisfying

$$A\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\2\\0\\4\end{bmatrix}, \qquad A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\-1\\-2\end{bmatrix}, \qquad A\begin{bmatrix}1\\0\\2\end{bmatrix} = \begin{bmatrix}-3\\2\\-1\\1\end{bmatrix}.$$

(Hint: Notice that e.g. $\begin{bmatrix}1\\0\\0\end{bmatrix} = 2\begin{bmatrix}1\\1\\1\end{bmatrix} - 2\begin{bmatrix}0\\1\\0\end{bmatrix} - \begin{bmatrix}1\\0\\2\end{bmatrix}$.)

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a) $\left[\begin{array}{ccc|ccc} 1&0&1&1&0&0\\ 2&1&2&0&1&0\\ 3&0&0&0&0&1\end{array}\right] \begin{array}{c} \\ -2(I) \\ -3(I)\end{array} \longrightarrow \left[\begin{array}{ccc|ccc} 1&0&1&1&0&0\\ 0&1&0&-2&1&0\\ 0&0&-3&-3&0&1\end{array}\right] :-3 \to \left[\begin{array}{ccc|ccc} 1&0&1&1&0&0\\ 0&1&0&-2&1&0\\ 0&0&1&1&0&-1/3\end{array}\right] \begin{array}{c} -(III) \\ \\ \\ \end{array} \to \left[\begin{array}{ccc|ccc} 1&0&0&0&0&1/3\\ 0&1&0&-2&1&0\\ 0&0&1&1&0&-1/3\end{array}\right]$

$$A^{-1} = \begin{bmatrix} 0 & 0 & 1/3 \\ -2 & 1 & 0 \\ 1 & 0 & -1/3 \end{bmatrix}$$

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b) $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\1\\-1\\-2\end{bmatrix}$

$A\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = A\left(2\begin{bmatrix}1\\1\\1\end{bmatrix} - 2\begin{bmatrix}0\\1\\0\end{bmatrix} - \begin{bmatrix}1\\0\\2\end{bmatrix}\right)$

$A\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = 2A\begin{bmatrix}1\\1\\1\end{bmatrix} - 2A\begin{bmatrix}0\\1\\0\end{bmatrix} - A\begin{bmatrix}1\\0\\2\end{bmatrix}$

$= 2\begin{bmatrix}1\\2\\0\\4\end{bmatrix} - 2\begin{bmatrix}0\\1\\-1\\2\end{bmatrix} - \begin{bmatrix}-3\\2\\-1\\1\end{bmatrix}$

$= \begin{bmatrix}2\\4\\6\\8\end{bmatrix} - \begin{bmatrix}0\\2\\-2\\4\end{bmatrix} - \begin{bmatrix}-3\\2\\-1\\1\end{bmatrix}$

$A\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}5\\0\\3\\3\end{bmatrix}$

$A\begin{bmatrix}1\\0\\2\end{bmatrix} = A\begin{bmatrix}1\\0\\0\end{bmatrix} + 2A\begin{bmatrix}0\\0\\1\end{bmatrix}$

$2A\begin{bmatrix}0\\0\\1\end{bmatrix} = A\begin{bmatrix}1\\0\\2\end{bmatrix} - A\begin{bmatrix}1\\0\\0\end{bmatrix}$

$= \begin{bmatrix}-3\\2\\-1\\1\end{bmatrix} - \begin{bmatrix}5\\0\\3\\3\end{bmatrix}$

$2A\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-8\\2\\-4\\-2\end{bmatrix}$

$A\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-4\\1\\-2\\-1\end{bmatrix}$

$$A = \begin{bmatrix} 5 & 0 & -4 \\ 0 & 1 & 1 \\ 3 & -1 & -2 \\ \cancel{3} & \cancel{2} & \cancel{-1} \end{bmatrix}$$

3. (10 points) (a) Find the matrix of reflection about the line $4y = 3x$ in $\mathbb{R}^2$.

(b) Describe the kernel of this matrix geometrically.

(c) Is the set of vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ satisfying $4y = 3x^2$ a subspace of $\mathbb{R}^2$? Justify your answer.

a) $y = \frac{3x}{4}$

$\vec{v} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$\vec{u}_v = \frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

matrix of projection $= \frac{1}{25}\begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix}$

matrix of reflection $= \begin{bmatrix} 7/25 & 24/25 \\ 24/25 & -7/25 \end{bmatrix}$

Reflection $= 2\text{proj}_{\vec{x}} - \vec{x}$

$= 2\frac{1}{25}\begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\left(2\frac{1}{25}\begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

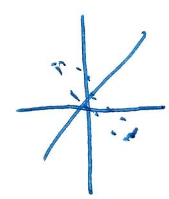
$= \vec{x}\left(\frac{1}{25}\begin{bmatrix} 32 & 24 \\ 24 & 18 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

$= \vec{x} \cdot \begin{bmatrix} \frac{32}{25} - 1 & 24/25 \\ 24/25 & 18/25 - 1 \end{bmatrix}$

$= \vec{x} \cdot \begin{bmatrix} 7/25 & 24/25 \\ 24/25 & -7/25 \end{bmatrix}$

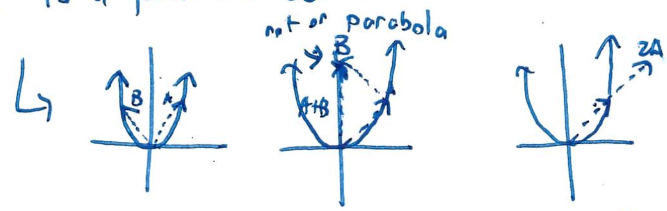
b) The Kernal is the set of vectors in a matrix domain that maps to $\vec{0}$.

The kernal of the reflection about a line through the origin is just ~~the~~ $\{0\}$, because vectors in generally chose direction, but not magnitude when reflected.



c) No, subspaces need to be linear and be closed under addition & scalar multiplication.

$y = \frac{3}{4}x^2$ is a parabola so it is not closed under addition & scalar multiplication.



A+B not on parabola 2A not on parabola

J.F

4. (10 points) (a) Identify the redundant vectors in the following list:

$$\begin{bmatrix}-1\\1\\3\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-2\\-3\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\4\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$$

(b) Consider the matrix $A = \begin{bmatrix}1 & 1\\0 & 2\end{bmatrix}$. Compute A^{2016}.

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not $\vec{0}$? ✓ not multiple of v_1 ✓

a) $\begin{bmatrix}-1\\1\\3\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-2\\3\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}$

X red $\vec{0}$

$\begin{bmatrix}-1 & -1 & 1\\1 & -2 & 1\\3 & -3 & 1\end{bmatrix}$

$$\begin{bmatrix}-1 & 1 & 3\\-1 & -2 & -3\\1 & 1 & 1\\0 & 2 & 4\\0 & 0 & 1\end{bmatrix} \div -1 \rightarrow \begin{bmatrix}1 & -1 & -3\\-1 & -2 & -3\\1 & 1 & 1\\0 & 2 & 4\\0 & 0 & 1\end{bmatrix} \begin{matrix}+(I)\\-(I)\end{matrix}$$

$$\begin{bmatrix}1 & -1 & -3\\0 & -3 & -6\\0 & 2 & 4\\0 & 2 & 4\\0 & 0 & 1\end{bmatrix} \begin{matrix}\div -3\\ \xrightarrow{-(III)}\end{matrix} \begin{bmatrix}1 & -1 & -3\\0 & 1 & 2\\0 & 2 & 4\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix} \xrightarrow{-2(II)} \begin{bmatrix}1 & -1 & -3\\0 & 1 & 2\\0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 1\end{bmatrix}$$

✓ redundant: $\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\2\\4\end{bmatrix}$

better to write in ~~...~~ 3x6 form

nonredundant: $\begin{bmatrix}-1\\1\\3\end{bmatrix}, \begin{bmatrix}-1\\-2\\-3\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

$A = \begin{bmatrix}1 & 1\\0 & 2\end{bmatrix} \begin{bmatrix}1 & 1\\0 & 2\end{bmatrix}$ 2^1

b) $A^2 = \begin{bmatrix}1 & 3\\0 & 4\end{bmatrix}$ 2^2

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$A^3 = \begin{bmatrix}1 & 7\\0 & 8\end{bmatrix}$ 2^3

$A^4 = \begin{bmatrix}1 & 15\\0 & 16\end{bmatrix}$ 2^4

$A^5 = \begin{bmatrix}1 & 31\\0 & 32\end{bmatrix}$

$A^a = \begin{bmatrix}1 & 2^a - 1\\0 & 2^a\end{bmatrix}$

$A^{2016} = \begin{bmatrix}1 & 2^{2016} - 1\\0 & 2^{2016}\end{bmatrix}$