Fall 2021 33A-2 Second Midterm

Full Name: ______________________________

UIN: ______________________________

Circle the name of your TA and the day of your discussion:

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Tuesday　　Thursday

Instrucitons

- Read each problem carefully.
- To get credit for a problem, you must show all of your reasoning and calculations.
- Box your final answer.
- No calculators are allowed.
- You may use the blank pages 3, 5, 7, 9 for scratch work. Work found on those pages will not be graded unless clearly indicated in the exam.
- **There will be 5 problems in the exam.** Problem 5 is True or False.

Problem	Points	Scores
1	15	
2	15	
3	15	
4	15	
5	10	
Total	70	

Problem 1 (15 points)**.** Consider a 3×3 matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 4 & 15 \\ 2 & 0 & -2 \end{bmatrix}.$$

1. (5 points) Compute the reduced row echelon form of A by the Gauss-Jordan elimination method.
2. (5 points) Find a basis of $\mathrm{im}(A)$ and compute $\dim(\mathrm{im}(A))$.
3. (5 points) Find a basis of $\ker(A)$ and compute $\dim(\ker(A))$.

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Problem 2 (15 points)**.** Let

$$\vec{v_1} = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}, \vec{v_2} = \begin{bmatrix} -\sqrt{3} \\ 1 \end{bmatrix}$$

and consider the basis $\mathcal{B} = \{\vec{v_1}, \vec{v_2}\}$ of $\mathbb{R}^2$.

1. (5 points) If $\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, compute $[\vec{x}]_{\mathcal{B}}$.

2. (5 points) If $[\vec{y}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$, compute $\vec{y}$.

3. (5 points) If $A = \frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$, compute the $\mathcal{B}$-matrix of A.

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Problem 3 (15 points)**.** Consider a 4×2 matrix

$$A = \begin{bmatrix} 1 & 1 \\ 7 & -5 \\ 1 & -3 \\ 7 & -9 \end{bmatrix}.$$

1. (5 points) Find an orthonormal basis of $\text{im}(A)$ by the Gram-Schmidt process.
2. (5 points) Find the QR factorization of A.
3. (5 points) Compute $\text{proj}_V \vec{b}$, where $V = \text{im}(A)$ and $\vec{b} = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

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Problem 4 (15 points)**.** Compute the following determinants:

$$1.\ \det\begin{bmatrix} 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 5 & 0 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \quad 2.\ \det\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad 3.\ \det\begin{bmatrix} 1 & 1 & -1 & -1 \\ 3 & 0 & 1 & 0 \\ -1 & -2 & 1 & 3 \\ 1 & 0 & 2 & 0 \end{bmatrix}.$$

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Problem 5 (10 points)**.** Answer True or False for each of the following statements. No justification is needed.

1. The standard basis $\{\vec{e_1}, \cdots, \vec{e_5}\}$ is a unique basis of $\mathbb{R}^5$.
2. Let $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation. Then $\dim(\ker(T)) \geq 1$.
3. An $n \times n$ matrix is orthogonal if the column vectors of the matrix are orthonormal.
4. The orthogonal complement of the orthogonal complement of a subspace V of $\mathbb{R}^m$ is V: $(V^{\perp})^{\perp} = V$.
5. We have $\det(A + B) = \det(A) + \det(B)$ for any 2×2 matrices A, B.