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Quiz 3
Math 33A-001
(Tuesday)
(10 minutes)

Show your work to receive partial credits. Use of calculator is not allowed for this quiz.

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1. 10 points Find a 2×2 matrix A such that $A \neq I_2$ but $A^2 = I_2$, where $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(Hint: Think geometrically!)

Let $T(\vec{x}) = A\vec{x}$ be a transformation.

Thus $T(T(\vec{x})) = A(A\vec{x}) = A^2\vec{x}$. For $T(T(\vec{x})) = I_2\vec{x} = \vec{x}$, the transformation must 'reverse' itself. T can be a rotational transformation of π radian, thus $T(T(\vec{x}))$ would rotate 2π radians, essentially not modifying $\vec{x}$.

Thus if $A = \begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

is a valid 2x2 matrix to fulfill conditions.

Proof: $A \cdot A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1\cdot-1+0\cdot0 & -1\cdot0+0\cdot-1 \\ 0\cdot-1+-1\cdot0 & 0\cdot0+-1\cdot-1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$