Midterm 1
Linear Algebra and Applications
(Math 33A-001)

Answer the questions in the space provided. If you run out of room for an answer, continue on the back of the page. **Show your work.**

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Question:	1	2	3	4	Total
Points:	5	5	5	5	20
Score:	5	5	3	5	18

1. $\boxed{\text{5 points}}$ Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that

$$T\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1\\-1\\0\end{pmatrix},\quad T\begin{pmatrix}-1\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\2\\-6\end{pmatrix}\quad\text{and}\quad T\begin{pmatrix}-1\\-1\\2\end{pmatrix}=\begin{pmatrix}-1\\0\\4\end{pmatrix}.$$

Compute the matrix of T.

$$T=\begin{pmatrix}1 & \vdots & \vdots\\ -1 & v_2 & v_3\\ 0 & \vdots & \vdots\end{pmatrix}\qquad T\begin{pmatrix}-1\\1\\-1\end{pmatrix}=-\begin{pmatrix}1\\-1\\0\end{pmatrix}+\begin{pmatrix}v_2\end{pmatrix}-\begin{pmatrix}v_3\end{pmatrix}=\begin{pmatrix}3\\2\\-6\end{pmatrix}$$

$$=\begin{pmatrix}-1\\1\\0\end{pmatrix}+v_2 - v_3$$

$$T\begin{pmatrix}-1\\-1\\2\end{pmatrix}=\begin{pmatrix}-1\\1\\0\end{pmatrix}-v_2+2v_3=\begin{pmatrix}-1\\0\\4\end{pmatrix}$$

$v_{2x_1}-v_{3x_1}-1=3 \Rightarrow v_{2x_1}=v_{3x_1}+4 \rightarrow v_{3x_1}=4$

$2v_{3x_1}-v_{2x_1}-1=-1 \qquad 2v_{3x_1}-v_{3x_1}-4-1=-1 \qquad v_{2x_1}=8$

$v_{2x_2}-v_{3x_2}+1=2 \rightarrow v_{2x_2}=v_{3x_2}+1 \rightarrow v_{3x_2}=0$

$2v_{3x_2}-v_{2x_2}+1=0 \qquad 2v_{3x_2}-v_{3x_2}+1=0 \qquad v_{2x_2}=1$

$v_{2x_3}-v_{3x_3}+0=-6 \rightarrow v_{2_3}=v_{3x_3}-6 \rightarrow v_{3x_3}=-2$

$2v_{3x_3}-v_{2x_3}=4 \qquad 2v_{3x_3}-v_{3x_3}+6=4 \qquad v_{2x_3}=-8$

$$T(\vec{x})\begin{pmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -8 & -2\end{pmatrix}\vec{x}\qquad \begin{bmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -2 & 8\end{bmatrix} \text{ is the matrix}$$

$$\boxed{\begin{bmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -8 & -2\end{bmatrix}} \leftarrow \text{ANSWER}\qquad \boxed{\begin{bmatrix}1 & 8 & 4\\ -1 & 1 & 0\\ 0 & -2 & 8\end{bmatrix}}$$

2. $\boxed{\text{5 points}}$ Find a 3×3 matrix A such that $A\vec{x}$ is parallel to the vector $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ for all $\vec{x} \in \mathbb{R}^3$.

Find proj matrix along $\vec{w} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$|\vec{w}| = \sqrt{1+1+1} = \sqrt{3}$

$\vec{u} = \left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

unit vec along line L

$\mathrm{Proj}_L = (\vec{x} \cdot \vec{u})\vec{u} = (x_1u_1 + x_2u_2 + x_3u_3)\vec{u}$

$= \begin{pmatrix} x_1u_1^2 + x_2u_1u_2 + x_3u_1u_3 \\ x_1u_1u_2 + x_2u_2^2 + x_3u_2u_3 \\ x_1u_1u_3 + x_2u_2u_3 + x_3u_3^2 \end{pmatrix}$

$A = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$ ← answer

$A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$

3. $\boxed{\text{5 points}}$ Let A be a 4×4 matrix, $\vec{b}$ is a non-zero vector in $\mathbb{R}^4$, and $\vec{0}$ is the zero vector in $\mathbb{R}^4$. We are told that the linear system $A\vec{x} = \vec{0}$ has ***infinitely*** many solutions. What can you say about the number of solutions of the system $A\vec{x} = \vec{b}$? You must explain your answer.

If $A\vec{x} = \vec{0}$ has ∞ solutions, rank(rrefA) < 4

so rref A looks smth like:

3

~~4~~

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

or any variant of a 4x4 rref matrix w/ 3 pivots, w/ at least one row of all zeroes

Thus $A\vec{x} = 0 \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ has infinite solutions.

answer → However, $A\vec{x} = \vec{b}$ has either no solutions or infinite solutions, dependent on if ~~the~~ $\vec{b}$ has a nonzero component in ~~the~~ a ~~same~~ row w/ no leading 1 (thus a row of all zeroes), as that creates an inconsistent row with $0 = b_i$ where $b_i \neq 0$.

In given example, $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ would have infinite solutions since $b_4 = 0$, so $0 = 0$, but $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ would have no solutions since the 4th row simplifies to $0 = 1$, inconsistent.

4. $\boxed{\text{5 points}}$ Let A be a $m \times n$ matrix and B a $n \times p$ matrix. If $\ker(A) = \{\vec{0}\}$ and $\ker(B) = \{\vec{0}\}$, then prove that $\ker AB = \{\vec{0}\}$.

Let $T(\vec{x}) = AB\vec{x}$

$\text{kern} AB = \{\vec{x} \mid AB\vec{x} = \vec{0}\}$

Let $AB\vec{x} = \vec{0} = A(B\vec{x})$ and let $B\vec{x} = \vec{c}$

thus $A\vec{c} = \vec{0}$

For $A\vec{c} = \vec{0}$, $\vec{c}$ must be an element of $\ker A$.

Thus for $A\vec{c} = \vec{0}$, $\vec{c} \in \{\vec{0}\}$ ~~always~~, using $\ker A = \{\vec{0}\}$

Thus since $\vec{c} \in \{\vec{0}\}$, $B\vec{x} \in \{\vec{0}\}$ ~~always~~.

For $B\vec{x} \in \{\vec{0}\}$, $\vec{x} \in \ker B \rightarrow \vec{x} \in \{\vec{0}\}$

thus $\vec{x} \in \{\vec{0}\}$ for $AB\vec{x} = \vec{0}$

Thus $\ker AB = \{\vec{0}\}$ since $\vec{0}$ is the only vector that causes $AB\vec{x}$ to equal $\vec{0}$

since $x \in \{\vec{0}\}$
For $AB\vec{x} = \vec{0}$

10/10

Quiz 3
Math 33A-001
(Tuesday)
(10 minutes)

Show your work to receive partial credits. Use of calculator is not allowed for this quiz.

Name: Christian Loanzon U ID: 005 109 250

TA's Name: Albert TA Meeting Day: Tues

1. $\boxed{\text{10 points}}$ Find a 2×2 matrix A such that $A \neq I_2$ but $A^2 = I_2$, where $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
(Hint: Think geometrically!)

Let $T(\vec{x}) = A\vec{x}$ be a transformation.
Thus $T(T(\vec{x})) = A(A\vec{x}) = A^2\vec{x}$. For $T(T(\vec{x})) = I_2\vec{x} = \vec{x}$,
the transformation must 'reverse' itself. T can be
a rotational transformation of π radian, thus $T(T(\vec{x}))$
would rotate 2π radians, essentially not modifying $\vec{x}$.

Thus if $A = \begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

is a valid 2×2 matrix to fulfill conditions.

Proof: $A \cdot A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1\cdot-1+0\cdot0 & -1\cdot0+0\cdot-1 \\ 0\cdot-1+-1\cdot0 & 0\cdot0+-1\cdot-1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$