Math 33A **Name:** ____________________

ID Number ____________________

Spring 2020
Midterm 2

This exam contains 9 pages (including this cover page) and 4 problems.

You are required to show your work on each problem on this exam. This holds even if work isn't explicitly asked in the statement of the problem. Please take care to show your steps, providing a reasonable amount of justification for your work that demonstrates your understanding. The following rules apply:

- You don't need to print this out and write directly on the exam, you may submit separate sheets of paper. If you print out and write on the exam, you may use the provided blank pages for your work. In either case, make sure you organize your work, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- In problems that have multiple parts, if your answer for a later part depends on a previous part, you can still get partial credit for the later part even if your answer for the previous part is incorrect.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Consider the matrix 4x3 matrix A with linearly independent columns

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

   (a) (5 points) Let $\mathcal{A}$ be the basis of im(A) consisting of the columns of A. Find the orthonormal basis $\mathcal{B}$ obtained by applying Gram-Schmidt to $\mathcal{A}$.

   (b) (5 points) Use the previous part to write down the QR factorization of A, being careful to justify your work.

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2. (a) (6 points) Suppose that A is a $3x3$ matrix given by

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

Let $\mathcal{B}$ be the basis of $\mathbb{R}^3$ given by

$$\mathcal{B} = \{(1,0,1),(1,0,0),(0,1,0)\}.$$

Find the $\mathcal{B}$-matrix of A; that is find the matrix B satisfying

$$[T(\vec{v})]_\beta = B[\vec{v}]_\beta$$

for all $\vec{v}$ in $\mathbb{R}^3$. Show all your steps.

(b) (4 points) (Unrelated to part (a)). Suppose that A is an mxn matrix corresponding to a linear transformation that is injective, and suppose $\vec{b}$ is a vector contained in $(im(A))^\perp$. Show that the least squares solution $\vec{x}^*$ to the system $A\vec{x} = \vec{b}$ is $\vec{x}^* = \vec{0}$.

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3. (a) (5 points) Let A be the 100x100 matrix that has all -1's below the diagonal, is 1 for every entry in the first row, and is 0 for all diagonal entries except for the first, depicted below.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \cdots & 1 & 1 \\ -1 & 0 & 0 & 0 \cdots & 0 & 0 \\ -1 & -1 & 0 & 0 \cdots & 0 & 0 \\ -1 & -1 & -1 & 0 \cdots & 0 & 0 \\ \vdots & & & & \vdots & \vdots \\ -1 & -1 & -1 & -1 \cdots & 0 & 0 \\ -1 & -1 & -1 & -1 \cdots & -1 & 0 \end{pmatrix}$$

Find the determinant of the matrix A, being sure to justify your work. (Hint: There is a trick that makes this much easier. Are there certain things we can do to a matrix that preserve determinants?)

(b) (5 points) Let A be an nxn matrix such that $A^m = 0$ for some positive integer m. (Here the 0 denotes the zero matrix, i.e the nxn matrix that has 0 in every entry, and the notation A^m means m copies of A multiplied together.) Prove that A is noninvertible.

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4. Let A be the matrix

$$\begin{pmatrix} 0 & 2 & 1 \\ 1 & 4 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

(a) (3 points) Find the eigenvalues of A.

(b) (3 points) Find the corresponding eigenspaces of A. List the algebraic and geometric multiplicities of each eigenvalue.

(c) (4 points) Explain why part b) tells you that A is diagonalizable. Diagonalize A by finding S, B so that $A = SBS^{-1}$ and B is diagonal.

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