Math 33A **Name:** ____________________

ID Number ____________________

Spring 2020
Midterm 1
04/24/20

This exam contains 9 pages (including this cover page) and 4 problems.

You are required to show your work on each problem on this exam. The following rules apply:

- You don't need to print this out and write directly on the exam, you may submit separate sheets of paper. If you print out and write on the exam, you may use the provided blank pages for your work. In either case, make sure you organize your work, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- In problems that have multiple parts, if your answer for a later part depends on a previous part, you can still get partial credit for the later part even if your answer for the previous part is incorrect.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Consider the following system of linear equations.

$$x_1 + x_2 + x_3 = 1$$

$$x_1 + x_2 + 2x_3 = 2$$

$$x_2 + x_4 = 2$$

   (a) (4 points) Rewrite the system as a matrix equation $A\vec{x} = \vec{b}$ and convert the augmented matrix $[A \,|\, \vec{b}]$ to reduced row echelon form.

   (b) (3 points) Is the linear transformation T corresponding to A injective? Surjective? Invertible?

   (c) (3 points) Write down the solution set to the linear system.

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2. Let A be the 3×3 matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 0 & 3 & 1 \end{pmatrix}.$$

(a) (4 points) Find a basis for the kernel of the corresponding linear transformation.

(b) (3 points) Find a basis for the image of the corresponding linear transformation.

(c) (3 points) Geometrically describe the kernel and the image.

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3. (a) (5 points) Let $\vec{e_1}, \vec{e_2}, \vec{e_3}$ be the standard basis vectors in $\mathbb{R}^3$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation that maps e_1 to e_2, maps e_2 to e_1, and maps the vector $(1, 0, 1)$ to e_3. Write down a matrix A such that T acts by multiplication by A on the left, i.e. $T(\vec{v}) = A\vec{v}$ for all $\vec{v} \in \mathbb{R}^3$.

   (b) (5 points) (This part is unrelated to part (a).) Write down a basis for the solution set to the linear system consisting of just the single equation

$$x_1 + 2x_2 + 3x_3 = 0.$$

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4. Let A be an $n \times m$ matrix, and let $\vec{b}$ be a vector in $\mathbb{R}^n$. Let $V_{\vec{b}}$ denote the solution set to $A\vec{x} = \vec{b}$. That is, we let

$$V_{\vec{b}} = \{\vec{x} \in \mathbb{R}^m |\ A\vec{x} = \vec{b}\}.$$

(We read the above as "the set of all $\vec{x}$ contained in $\mathbb{R}^m$ such that $A\vec{x} = \vec{b}$.")

(a) (3 points) Prove that $V_{\vec{b}}$ fails all three criteria for being a subspace if $\vec{b} \neq \vec{0}$.

(b) (3 points) Prove that if $V_{\vec{b}}$ is not the empty set, i.e. if $V_{\vec{b}}$ contains at least one vector denoted $\vec{x}$, then

$$V_{\vec{b}} = \vec{x} + \ker(A),$$

where $\vec{x} + \ker(A) = \{\vec{x} + \vec{y} |\ y \in \ker(A)\}$. In other words, $V_{\vec{b}}$ is equal to $\ker(A)$ shifted by the vector $\vec{x}$.

(c) (2 points) If $\ker(A)$ is a line, what is the size of a basis of $\text{im}(A)$? That is, what is the number of vectors in a basis for $\text{im}(A)$? (Hint: your answer could possibly depend on n and/or m).

(d) (2 points) If $\ker(A)$ is a line, is A injective? Surjective? Invertible? For each of these answer either "always", "sometimes" or "never."

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