MATH 33A – SECTION 2
MIDTERM #1

OCTOBER 23, 2015

Full Name	
Student ID	
Discussion Section	

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Problem 1. Determine whether there exists a polynomial of the form

$$f(t) = a + bt + ct^2 + dt^3$$

whose graph goes through the points $(0, 1)$, $(1, 0)$, $(-1, 0)$, $(2, -15)$, $(-2, 9)$ and $(3, -56)$. In the affirmative case, give $f(t)$.

Problem 2. Consider the matrix

$$A_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Recall that the linear transformation $R_\theta : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by $R_\theta(\vec{x}) = A_\theta \vec{x}$ is a counter-clockwise rotation through an angle θ.

(a) Show that

$$R_\alpha \circ R_\beta = R_\beta \circ R_\alpha = R_{\alpha+\beta}$$

by calculation.

(b) Using the result of (**a**), show that R_α is invertible and describe R_α^{-1}.

Hint: For (**a**), you may use the trigonometric identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

Problem 3. Determine all the values (if any) of the constants B and C for which the following matrix is invertible:

$$\begin{bmatrix} 0 & 1 & B \\ -1 & 0 & C \\ -B & -C & 0 \end{bmatrix}$$

Problem 4. Let $A = \begin{bmatrix} 4 & 8 & 1 & 1 & 4 \\ 3 & 6 & 1 & 2 & 5 \\ 2 & 4 & 1 & 9 & 10 \\ 1 & 2 & 3 & 2 & 0 \end{bmatrix}$.

(**a**) Find a basis for $\ker(A)$.

(**b**) Find a basis for $\text{im}(A)$.