Math 33A – Midterm 2

Your name: Usama Riaz, 8044476

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There are four problems on this exam, each worth ten points. Good luck!

39/40

5

Problem 1: **(a)** Which of these lists of vectors have the same spans?

$$A = \left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}, \cancel{\begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}} \right) \quad B = \left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right) \quad C = \left(\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \right)$$

$$D = \left(\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right) \quad E = \left(\begin{bmatrix} -4 \\ -2 \\ 6 \end{bmatrix}, \cancel{\begin{bmatrix} 6 \\ 3 \\ -9 \end{bmatrix}} \right) \quad F = \left(\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \right)$$

$-6 \cdot \frac{6}{4} = \frac{-36}{4} = -9$

~~span(A) =~~ $\left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix} \right)$

span(A) = span(F)

span(D) = span(B)

span(E) = span(C)

5

(b) Which of these lists of vectors are linearly independent?

$$\cancel{A} = \left(\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \cancel{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \right) \quad B = \left(\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right)$$

$$C = \left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right) \quad D = \left(\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \cancel{\begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}} \right)$$

$\boxed{B, C}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & -1 & 3 \end{bmatrix}$$

2

9

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Problem 2:(a) Find a basis for the kernel and a basis for the image, for the following matrix:

4

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & 3 \end{bmatrix}.$$

ker(A) = rref(A)

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 = x_3, \quad x_2 = \cancel{x_3} - 2x_3$

$$\begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

~~4x~~

$$\boxed{\text{im}(A) \rightarrow \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}$$

✓

let $\cancel{t}$ 1,

basis(A) → $\boxed{\cancel{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}}$

5

(b) Let P be a plane through the origin in R^3, and let $T : R^3 \rightarrow R^3$ be orthogonal projection onto P. Describe the kernel and image of T.

ker(T) is the set of vectors orthogonal to P.

im(T) is P.

10

$v_1 = [2,1,-3]$ $v_2 = [1,2,-3]$

Problem 3:(a) Let P be the set of points in R^3 satisfying $x + y + z = \cancel{3}$ 0 and let $v_1 = \cancel{[2,1,0]}$, $v_2 = \cancel{[1,2,0]}$. Then $\{v_1, v_2\}$ is a basis for P. Use the Gram-Schmidt process to find an orthonormal basis for P, starting with $\{v_1, v_2\}$.

$$e_{v_1} = \frac{\langle 2,1,-3\rangle}{\sqrt{4+1+9}} = \boxed{\frac{\langle 2,1,-3\rangle}{\sqrt{14}}}$$

$$v_2^{\parallel} = (v_2 \cdot e_{v_1}) e_{v_1} = \left(\langle 1,2,-3\rangle \cdot \frac{\langle 2,1,-3\rangle}{\sqrt{14}}\right)\frac{\langle 2,1,-3\rangle}{\sqrt{14}}$$

$$= \frac{13}{14}\langle 2,1,-3\rangle$$

$$v_2^{\perp} = \langle 1,2,-3\rangle - \left\langle \frac{26}{14}, \frac{13}{14}, \frac{-39}{14}\right\rangle = \left\langle -\frac{6}{7}, \frac{15}{14}, \frac{-3}{14}\right\rangle =$$

$\frac{13}{7}$

$$v_2^{\perp} = -\frac{3}{14}\langle 4,-5,1\rangle \rightarrow \boxed{e_{v_2} = \frac{-\frac{3}{14}\langle 4,-5,1\rangle}{\sqrt{52}}}$$

225
+144
369
+9
378

(b) The vectors $v_1 = [1/\sqrt{2}, -1/\sqrt{2}, 0]$, $v_2 = [0,0,1]$ form an orthonormal basis for the plane $x = -y$. Use this basis to find the orthogonal projection of the vector $[\cancel{3,}4]$ onto this plane.

$[3,4,0]$

$$\left(\langle 3,4,0\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\rangle\right)\left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right\rangle$$

$$= \left(\frac{3}{\sqrt{2}} - \frac{4}{\sqrt{2}}\right)\left(\left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right\rangle\right) = \frac{-1}{\sqrt{2}}\left\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right\rangle$$

$$\left(\langle 3,4,0\rangle \cdot \langle 0,0,1\rangle\right)\left(\langle 0,0,1\rangle\right)$$

$$= 0$$

$$\mathrm{proj}_P\,[3,4,0] = \left\langle -\frac{1}{2}, \frac{1}{2}, 0\right\rangle$$

$3(3-4) - 1(3-6)$ 10

$-1(2-3)$

$3(-1) - (-3) - (-1)$

$-3 + 3 + 1$

Problem 4:(a) What is the determinant of the following matrix?

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 2 \\ 3 & 2 & 3 \end{bmatrix}$$

$3(3-4) - (3-6) - 1(2-3)$

$= -3 + 3 + 1 = \boxed{1}$ ✓

(b) Consider the matrix

$$B = \begin{bmatrix} 2-t & 1 \\ 1 & 2-t \end{bmatrix}.$$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

For which values of t is this matrix **not** invertible?

$ad - bc = 0$

$2 - (-1) = 3$

$(2-t)^2 - 1 = 0$

$(2-t)(2-t)$

$= 4$

$4 - 4t + t^2 - 1 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$\boxed{t = 3 \text{ and } t = 1}$ ✓