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Math 33A Quiz 8

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SECTION: Cross one box below

Day \ T.A.	Bon-Soon	David	Robert
Tuesday	2A	2C	2E
Thursday	2B	2D	~~2F~~

Instructions:
Solve the problem
You have 8 minutes
Use a pen to record your final answer

Problem 1. Let A be the matrix be the matrix $A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix}$. Find the eigenvalues of A and determine whether it is diagonalizable.

$(-\lambda)\left((\lambda)(-2-\lambda)+1\right)$

$(-\lambda)(\lambda^2+2\lambda+1)$

$\begin{matrix} -\lambda & 0 & 0 \\ 1 & -\lambda & -1 \\ 0 & 1 & -2-\lambda \end{matrix}$ $(-\lambda)(\lambda+1)(\lambda+1)$

$\begin{matrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{matrix}$ $\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{matrix}$

$f_A(\lambda) = \det(A-\lambda I) = \det\begin{pmatrix} -\lambda & 0 & 0 \\ 1 & -\lambda & -1 \\ 0 & 1 & -2-\lambda \end{pmatrix} = (-\lambda)\left((\lambda)(-2-\lambda)+1\right)$

$= (-\lambda)\left(\lambda^2+2\lambda+1\right)$

$= (-\lambda)(\lambda+1)(\lambda+1)$

$= -(\lambda)(\lambda+1)^2$

$\lambda_1 = 0$ AM = 1.

$\lambda_2 = -1$ AM = 2

↳ these are the eigenvalues of A.

$E_{\lambda_2} = E_{-1} = \ker(A+I)$

$= \ker\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix} \to \ker\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{pmatrix}$

$= \ker\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

This matrix is rank 2, therefore

$\dim(\ker) = (3-2) = 1 = GM(\lambda_2) \neq AM(\lambda_2) = 2$

since $GM(\lambda_2) \neq AM(\lambda_2)$

A is Not diagonizable

$E_{\lambda_1} = E_0 = \ker(A-0I)$

$= \ker(A)$

$= \ker\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -2 \end{pmatrix} \to \ker\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

This matrix has rank 2, therefore

$\dim(\ker) = 1 = GM(\lambda_1) = GM(0) = AM(\lambda_1)$ good.