Math 33A-2 Exams

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TOTAL POINTS

23 / 24

QUESTION 1

1 5 / 6

- **0 pts** Correct
- **1 pts** Minor error in matrix like miscopying number
- **2 pts** Major error like flipping row and column or row reducing A
- **3 pts** No matrix written in part a
- ✓ **- 1 pts Error in row reduction**
- **2 pts** Major error in row reduction, tried to find inverse instead of solving system directly, or deduced inconsistent
- **3 pts** No row reduction

QUESTION 2

6 pts

2.1 3 / 3

- ✓ **- 0 pts Correct**
- **1 pts** rows in wrong order
- **1 pts** there are non-zeros above/below leading ones
- **1 pts** leading entries aren't 1

2.2 3 / 3

- ✓ **- 0 pts Correct**
- **2 pts** invalid row operation
- **3 pts** matrix is not in rref
- **2 pts** rref is not calculated
- **1 pts** dropped entry (possible typo)
- **1 pts** arithmetic error

QUESTION 3

6 pts

3.1 3 / 3

- **2 pts** Incorrect answer
- **1 pts** without doing row operations
- ✓ **- 0 pts correct**
- **1 pts** computational error
- **1 pts** computational error

3.2 3 / 3

- ✓ **- 0 pts Correct**
- **1 pts** without rref or incorrect rref or without a matrix with 0 row
- **1 pts** without explanation or incorrect explanation
- **3 pts** incorrect answer

QUESTION 4

4 6 / 6

- ✓ **- 0 pts Correct**
- **2 pts** order wrong
- **1 pts** proj not computed/computed incorrectly
- **1 pts** order not shown
- **0.5 pts** order half shown
- **1 pts** ref not computed/computed incorrectly
- **1 pts** ref formula missing
- **1 pts** proj formula missing
- **6 pts** incorrect
- **0 pts** perfect!

gradescope

MATH 33A-2, 1. MIDTERM (A)

For this exam the only permitted assistance is a handwritten 4×6 index card. Books, lecture notes, or any technical devices such as calculators, computers or phones are strictly prohibited!

Please, write your name, student ID number, and discussion section.

Name: John Arthur Dang
UID: [redacted]
Section: 2F

Problems	Points	Score
1	6	
2	6	
3	6	
4	6	
Total	24	

It is important that you show your work.

Problem 1 (6 points). Let $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 2x_3 + x_4 \\ x_2 + 4x_3 \\ \frac{1}{2}x_1 - \frac{1}{2}x_3 + 5x_4 \\ x_1 + 3x_2 \end{bmatrix}$$

a.) (3 points) Find the matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^4$.

b.) (3 points) Let

$$\vec{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -\frac{1}{2} \end{bmatrix}$$

be a vector in $\mathbb{R}^4$. Find the solution to $T(\vec{x}) = \vec{b}$, in other words, find a vector $\vec{x}$ in $\mathbb{R}^4$ such that $A\vec{x} = \vec{b}$.

a) $T(X) = A\vec{x} = \begin{bmatrix} 2x_1 + 0 - 2x_3 + x_4 \\ 0 + x_2 + 4x_3 + 0 \\ x_1/2 + 0 - x_3/2 + 5x_4 \\ x_1 + 3x_2 + 0 + 0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & -2 & 1 \\ 0 & 1 & 4 & 0 \\ 1/2 & 0 & -1/2 & 5 \\ 1 & 3 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

Thus, matrix $A = \begin{bmatrix} 2 & 0 & -2 & 1 \\ 0 & 1 & 4 & 0 \\ 1/2 & 0 & -1/2 & 5 \\ 1 & 3 & 0 & 0 \end{bmatrix}$

b) $A\vec{x} = \vec{b} \Rightarrow \begin{bmatrix} 2 & 0 & -2 & 1 \\ 0 & 1 & 4 & 0 \\ 1/2 & 0 & -1/2 & 5 \\ 1 & 3 & 0 & 0 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -\frac{1}{2} \end{bmatrix}$

can be written as the augmented matrix

$\left(\begin{array}{cccc|c} 2 & 0 & -2 & 1 & 1 \\ 0 & 1 & 4 & 0 & 2 \\ 1/2 & 0 & -1/2 & 5 & -1 \\ 1 & 3 & 0 & 0 & -1/2 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -1/2 \\ 0 & 1 & 4 & 0 & 2 \\ 1/2 & 0 & -1/2 & 5 & -1 \\ 2 & 0 & -2 & 1 & 1 \end{array}\right)$

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -1/2 \\ 0 & 1 & 4 & 0 & 2 \\ 1/2 & 0 & -1/2 & 5 & -1 \\ 2 & 0 & -2 & 1 & 1 \end{array}\right) \xrightarrow[-2(I)]{2\cdot} \left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -1/2 \\ 0 & 1 & 4 & 0 & 2 \\ 1 & 0 & -1 & 10 & -2 \\ 0 & -6 & -2 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{-(I)} \left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -1/2 \\ 0 & 1 & 4 & 0 & 2 \\ 0 & -3 & -1 & 10 & -3/2 \\ 0 & -6 & -2 & 1 & 2 \end{array}\right) \xrightarrow{\substack{-3(II) \\ +3(II) \\ +6(II)}} \left(\begin{array}{cccc|c} 1 & 0 & -12 & 0 & -13/2 \\ 0 & 1 & 4 & 0 & 2 \\ 0 & 0 & 11 & 10 & 9/2 \\ 0 & 0 & 22 & 1 & 14 \end{array}\right)$$

$$\xrightarrow{\substack{+(III) \\ -2(III)}} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 10 & -2 \\ 0 & 1 & 4 & 0 & 2 \\ 0 & 0 & 11 & 10 & 9/2 \\ 0 & 0 & 0 & -19 & 5 \end{array}\right) \xrightarrow{\div 11} \left(\begin{array}{cccc|c} 1 & 0 & -1 & 10 & -2 \\ 0 & 1 & 4 & 0 & 2 \\ 0 & 0 & 1 & 10/11 & 9/22 \\ 0 & 0 & 0 & -19 & 5 \end{array}\right)$$

$$\xrightarrow{\substack{+(III) \\ -4(III)}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{120}{11} & 3 \\ 0 & 1 & 0 & -\frac{40}{11} & 8/22 \\ 0 & 0 & 1 & 10/11 & 9/22 \\ 0 & 0 & 0 & -19 & 5 \end{array}\right) \xrightarrow{\div -19} \left(\begin{array}{cccc|c} 1 & 0 & 0 & \frac{120}{11} & 3 \\ 0 & 1 & 0 & -\frac{40}{11} & 8/22 \\ 0 & 0 & 1 & 10/11 & 9/22 \\ 0 & 0 & 0 & 1 & -5/19 \end{array}\right)$$

$$\xrightarrow{\substack{-\frac{120}{11}(IV) \\ +\frac{40}{11}(IV) \\ -\frac{10}{11}(IV)}} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3+\frac{120}{11}\left(\frac{5}{19}\right) \\ 0 & 1 & 0 & 0 & \frac{8}{22}-\frac{40}{11}\left(\frac{5}{19}\right) \\ 0 & 0 & 1 & 0 & \frac{9}{22}+\frac{10}{11}\left(\frac{5}{19}\right) \\ 0 & 0 & 0 & 1 & -5/19 \end{array}\right)$$

$$\vec{b} = \begin{bmatrix} 3+\frac{120(5)}{11(19)} \\ \frac{8}{22}-\frac{40}{11}\left(\frac{5}{19}\right) \\ \frac{9}{22}+\frac{10}{11}\left(\frac{5}{19}\right) \\ -5/19 \end{bmatrix}$$

Problem 2 (6 points). Are the following matrices in reduced row echelon form? If not, find the reduced row echelon form.

a.) (3 points) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 4 & \frac{1}{2} \end{bmatrix}$ → NO $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 4 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$ → ÷4 $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1/8 \\ 0 & 0 & 1 \end{bmatrix}$

→ $-2(\text{III})$, $-\frac{1}{8}(\text{III})$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b.) (3 points) $B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ NO

→ $-8(\text{III})$ $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Problem 3 (6 points). For each of the matrices below, either find an inverse or explain why no inverse exists.

a.) (3 points) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-3(I)]{-2(I)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 2 & 1 & -3 & 0 & 1 \end{array}\right] \xrightarrow{-2(II)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & -2 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

b.) (3 points) $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 7 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-(I)]{-(I)} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 3 & 6 & -1 & 0 & 1 \end{array}\right] \xrightarrow[-3(II)]{-(II)} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 0 & 3 & -3 & 1 \end{array}\right]$$

No inverse because $\text{rank}(B) = 2 < 3$

Problem 4 (6 points). Find a matrix which describes the projection on the horizontal line combined with a reflection about the vertical line in $\mathbb{R}^2$. Does the order of these transformations matter?

Projection on horizontal line is projection on vector $\vec{u} = \langle 1, 0 \rangle$ described by matrix

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Reflection about the vertical line is described by matrix

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

The transformation matrix is $\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

The order does not matter because

$$AB = BA = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

SCRATCH WORK

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