Math 33A-2 Exams2

Siddharth Joshi

TOTAL POINTS

23 / 24

QUESTION 1

6 pts

1.1 3 / 3

✓ **- 0 pts** Correct

- 2 pts invalid row operation

- 3 pts matrix is not in rref

- 2 pts rref is not calculated

- 1 pts dropped entry (possible typo)

- 1 pts arithmetic error

- 1 pts rows are in wrong order

- 2 pts rref calculated incorrectly

1.2 3 / 3

✓ **- 0 pts** Correct

- 1 pts rows in wrong order

- 1 pts there are non-zeros above/below leading ones

- 1 pts leading entries aren't 1

- 1 pts arithmetic error

QUESTION 2

2 6 / 6

✓ **- 0 pts** Correct

- 1 pts Minor error in matrix

- 2 pts Major error like flipping row and column or row reducing A

- 3 pts No matrix written in part a

- 1 pts Error in row reduction

- 2 pts Major error in row reduction, tried to find inverse instead of solving system directly, or deduced inconsistent

- 3 pts No row reduction

QUESTION 3

6 pts

3.1 3 / 3

- 2 pts Incorrect answer

- 1 pts without doing row operations

✓ **- 0 pts** correct

- 1 pts mistake answer

- 1 pts not computed inverse

3.2 3 / 3

✓ **- 0 pts** Correct

- 1 pts without rref or incorrect rref or without a matrix with 0 row

- 1 pts without explanation or incorrect explanation

- 3 pts incorrect answer

QUESTION 4

4 5 / 6

- 2 pts the derivation of the matrices for the concrete reflection and projection is missing

- 2 pts composition/ product and commutativity is wrong

- 1 pts reflection matrix computed incorrectly

- 2 pts missing formulas for projection and reflection

- 1 pts reflection matrix is not derived

- 1 pts commutativity is not shown

- 1 pts projection matrix is computed incorrectly

- 0 pts correct

- 0 pts perfect!

- 0.5 pts commutativity is half shown

✓ **- 1 pts projection matrix is computed incorrectly**

- 6 pts incorrect

- 1 pts projection matrix is not stated

- 2 pts formulas not given

- 1 pts missing formula reflection

MATH 33A-2, 1. MIDTERM (B)

For this exam the only permitted assistance is a handwritten 4×6 *index card. Books, lecture notes, or any technical devices such as calculators, computers or phones are strictly prohibited!*

Please, write your name, student ID number, and discussion section.

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Section: 2E

Problems	Points	Score
1	6	
2	6	
3	6	
4	6	
Total	24	

It is important that you show your work.

Problem 1 (6 points). Are the following matrices in reduced row echelon form? If not, find the reduced row echelon form.

a.) (3 points) $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

A is already in rref as all the ~~rows~~ with non-zero rows have leading 1s ~~have~~ 0s and the leadings 1s ~~don't~~ have columns

0s in ... and the leadings 1s

b.) (3 points) $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & \frac{1}{3} \\ 1 & 0 & 2 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1/3 \\ 1 & 0 & 2 \end{bmatrix} \xrightarrow{\text{swap row 1 and 3}}$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1/3 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{divide row 2 by 2}}$

Ans $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1/6 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{subtract } 2 \times \text{row 3 from row 1 and } 1/6 \times \text{row 3 from 2}}$

1b

Ans. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1. a) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{\text{subtract}}$ 6 x row 3 from row 2

Ans. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Problem 1 (6 points). Let $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} x_2 + 4x_3 \\ x_1 - x_3 + \frac{1}{2}x_4 \\ \frac{1}{2}x_1 - \frac{1}{2}x_3 + 5x_4 \\ x_1 + 3x_2 \end{bmatrix}$$

a.) (3 points) Find the matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^4$.
b.) (3 points) Let

$$\vec{b} = \begin{bmatrix} \frac{1}{2} \\ 2 \\ 1 \\ -1 \end{bmatrix}$$

be a vector in $\mathbb{R}^4$. Find the solution to $T(\vec{x}) = \vec{b}$, in other words, find a vector $\vec{x}$ in $\mathbb{R}^4$ such that $A\vec{x} = \vec{b}$.

a) $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 + 4x_3 \\ x_1 - x_3 + \frac{1}{2}x_4 \\ \frac{1}{2}x_1 - \frac{1}{2}x_3 + 5x_4 \\ x_1 + 3x_2 \end{bmatrix}$

$\Rightarrow$ ~~A~~ from row 1 of A : $A_{12} = 1$ $A_{13} = 4$

— " — 2 " : $A_{21} = 1$ $A_{23} = -1$ $A_{24} = \frac{1}{2}$

— " — 3 " : $A_{31} = \frac{1}{2}$ $A_{33} = -\frac{1}{2}$ $A_{34} = 5$

— " — 4 " : $A_{41} = 1$ $A_{42} = 3$

and the rest $A_{i,j}$ where $i,j \in \{1 \cdots 4\}$ are 0

$\Rightarrow A = \begin{pmatrix} 0 & 1 & 4 & 0 \\ 1 & 0 & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 5 \\ 1 & 3 & 0 & 0 \end{pmatrix}$

b) $A\vec{x} = \vec{b}$

$\Rightarrow \begin{pmatrix} 0 & 1 & 4 & 0 \\ 1 & 0 & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 5 \\ 1 & 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 2 \\ 1 \\ -1 \end{pmatrix}$

∴ the augmented matrix $[A \,\vdots\, \vec{b}]$

is $\left[\begin{array}{cccc|c} 0 & 1 & 4 & 0 & 1/2 \\ 1 & 0 & -1 & 1/2 & 2 \\ 1/2 & 0 & -1/2 & 5 & 1 \\ 1 & 3 & 0 & 0 & -1 \end{array}\right]$ ∴ compute rref$([A \,\vdots\, \vec{b}])$:

swap rows to get:

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -1 \\ 0 & 1 & 4 & 0 & 1/2 \\ 1 & 0 & -1 & 1/2 & 2 \\ 1/2 & 0 & -1/2 & 5 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -1 \\ 0 & 1 & 4 & 0 & 1/2 \\ 0 & -3 & -1 & 1/2 & 3 \\ 0 & -3/2 & -1/2 & 5 & 3/2 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -12 & 0 & -2.5 \\ 0 & 1 & 4 & 0 & 1/2 \\ 0 & 0 & 11 & 1/2 & 4.5 \\ 0 & 0 & 5.5 & 5 & 3/2+3/4 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 12/22 & -2.5+12\left(\frac{4.5}{11}\right) \\ 0 & 1 & 0 & -4/22 & 1/2-4\left(\frac{4.5}{11}\right) \\ 0 & 0 & 1 & 1/22 & 4.5/11 \\ 0 & 0 & 0 & \frac{104.5}{22} & \frac{3}{2}+\frac{3}{4}-2.25 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cccc|l} 1 & 0 & 0 & 0 & -2.5+12\left(\frac{4.5}{11}\right)-\frac{12}{104.5}\left(\frac{3}{2}+\frac{3}{4}-2.25\right) \\ 0 & 1 & 0 & 0 & 1/2-4\left(\frac{4.5}{11}\right)+\frac{4}{104.5}\left(\frac{3}{2}+\frac{3}{4}-2.25\right) \\ 0 & 0 & 1 & 0 & 4.5/11-\frac{1}{104.5}\left(\frac{3}{2}+\frac{3}{4}-2.25\right) \\ 0 & 0 & 0 & 1 & \frac{22}{104.5}\left(\frac{3}{2}+\frac{3}{4}-2.25\right) \end{array}\right]$$

$$\therefore \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2.5+12\left(\frac{4.5}{11}\right) \\ 0 & 1 & 0 & 0 & 1/2-4\left(\frac{4.5}{11}\right) \\ 0 & 0 & 1 & 0 & 4.5/11 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 53/22 \\ 0 & 1 & 0 & 0 & -25/22 \\ 0 & 0 & 1 & 0 & 9/22 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

∴

Ans. $\vec{x} = \begin{pmatrix} 53/22 \\ -25/22 \\ 9/22 \\ 0 \end{pmatrix}$

Problem 3 (6 points). For each of the matrices below, either find an inverse or explain why no inverse exists.

a.) (3 points) $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

compute rref $\left[\begin{array}{ccc|ccc} \multicolumn{3}{c|}{\overbrace{\quad\quad}^{A}} & \multicolumn{3}{c}{\overbrace{\quad\quad}^{I_3}} \\ 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$$\to \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \text{Ans. } A^{-1} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{\quad\quad\quad}_{I_3} \qquad \underbrace{\quad\quad\quad}_{A^{-1}}$$

b.) (3 points) $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 8 & 14 \\ 1 & 2 & 3 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 8 & 14 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 6 & 12 & -2 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & \cancel{6}\,1 & 2 & -1 & 1/2 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1/2 & 0 \\ 0 & 1 & 2 & -1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & -1/2 & 1 \end{array}\right]$$

rref$(B \,\vdots\, I_3)$ is not of the form $[I_3 \,\vdots\, B^{-1}]$ Hence no inverse exists for B

Problem 4 (6 points). Find a matrix which describes the projection on the vertical line combined with a reflection about the horizontal line in $\mathbb{R}^2$. Does the order of these transformations matter?

Let S: projection onto vertical line $(x=0)$ $(\mathbb{R}^2 \Rightarrow \mathbb{R}^2)$

Let $S(v) \equiv A v$ for all $v \in \mathbb{R}^2$

Let ~~T(v)~~ $T: \mathbb{R}^2 \Rightarrow \mathbb{R}^2$ be the reflection about the horizontal line $(y=0)$

$\therefore$ let B be a matrix s.t. $T(v) = Bv \ \forall v \in \mathbb{R}^2$

$\therefore$ ~~A~~ Let u_A be a unit vector in $x=0$

$u_a = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \therefore A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ using ~~formula~~ for matrix of projection

~~Let~~ Let u_b be a unit vector in $y=0$

$u_b = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \therefore \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$\therefore$ ~~As~~ The ~~projecti~~ composite transformation $T \circ S$ is represented by $B \times A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$

the composite transformation $S \circ T$ is represented by

$A \times B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$

since ~~$A\times$~~ $B\times A = A\times B$

the composite transformations ~~$S\circ T$~~ $T\circ S(v) = S\circ T(v)$

$\forall\ v\in\mathbb{R}^2$

$\therefore$ the order of these transformations doesn't matter

As $T\circ S$ represents transformation S followed by T

while $S\circ T$ – " ——— T followed by S

But these are equivalent (shown above)

Hence order <u>does</u> <u>not</u> <u>matter</u>.

SCRATCH WORK

SCRATCH WORK