Nan	ne_		UID_			
2A	Redmond McNamara	T	GEOLOGY 6704			
2B		R	ROYCE 154			
2C	Albert Zheng	T	ROLFE 3121	G - 1:	2	0
2D		R	ROYCE 162	Section	2	10
2E	Weiyi Liu	T	BOELTER 5280			
2F		R	BOELTER 5436			

Sign your name on the line below if you do NOT want your exam graded using GradeScope. Otherwise, keep it blank. If you sign here, we will grade your paper exam by hand and a) you will not get your exam back as quickly as everyone else, and b) you will not be able to keep a copy of your graded exam after you see it.

- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

Page:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

You may use this page for scratch work

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$$\vec{V}_{3} = \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ -4 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & -1 \\ -4 & 3 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & -1 \\ -4 & 3 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & -1 \\ \sqrt{2} & -1 \\ \sqrt{2} & -1 \\ \sqrt{2} & -1 \end{bmatrix}$$

$$\vec{V}_{3} \cdot \vec{U}_{1} = \begin{bmatrix} \frac{2}{3} + 2 - \frac{3}{2} + 1 \\ -\frac{2}{3} & -\frac{3}{3} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} - \frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} - \frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} - \frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} - \frac{7}{2} & -\frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} & -\frac{7}{2} \\ -\frac{7}{2} & -\frac{7}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{2} - \frac{3}{2} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} - \frac{1}{4} & -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} - \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} - \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \\ -\frac{1}{4} - \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} + \frac{1}{4} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1$$

1. (10 points) (a) Which of the following is the QR-factorization of the matrix

Circle your answer.

$$\begin{bmatrix} 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \\ -1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & -7 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1/2 \\ 0 & 3 & 5 \\ 0 & 0 & -7 \end{bmatrix} \begin{bmatrix} 2 & 0 & 12 \\ 0 & 1/\sqrt{2} & -1/2 \\ 1/\sqrt{2} & 0 & 1/2 \\ 0 & 1/\sqrt{2} & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 12 \\ 0 & 12 & 18 \\ 0 & 0 & 6\sqrt{2} \end{bmatrix}$$

$$3 + \frac{5}{2} \times \frac{7}{2} = Q$$

$$3 + \frac{5}{2} \times \frac{7}{2} = Q$$

$$1/2 & 1/2 & 1/2 \\ 1 & -4 & 22 \\ 1 & -4 & 29 \\ 1 & -5 & 37 \end{bmatrix} \begin{bmatrix} 1 & 6 & 10 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$$

$$1/2 & 1/2 & 1/2 \\ -1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 7 \end{bmatrix}$$

$$2 \times \frac{7}{2} \times \frac{7}{2} = Q$$

(b) Write down the result of the Gram-Schmidt process on the vectors

$$ec{v}_1 = egin{bmatrix} 1 \ -1 \ -1 \ 1 \end{bmatrix}, \quad ec{v}_2 = egin{bmatrix} 1 \ 2 \ 2 \ 1 \end{bmatrix}, \quad ec{v}_3 = egin{bmatrix} 9 \ -4 \ 3 \ 2 \end{bmatrix}.$$

You do not have to show your work (if you need more room, use Page 0).

$$\begin{array}{c} \sqrt{2} \\ \sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2} \\ \sqrt{2} \\ -\sqrt{2} \\ \end{array}$$

$$3+\frac{5}{2}+\frac{7}{2}=9$$
.
 $-3+\frac{5}{2}+\frac{7}{2}=9$.
 $3+\frac{5}{2}+\frac{7}{2}=9$.
 $3+\frac{5}{2}-\frac{7}{2}=9$.
 $-3+\frac{5}{2}-\frac{7}{2}=9$.
 $-3+\frac{5}{2}-\frac{7}{2}=9$.
 $-3+\frac{5}{2}-\frac{7}{2}=9$.
 $-3+\frac{5}{2}-\frac{7}{2}=9$.

2. (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}.$$

(a) Suppose that \vec{v} is any nonzero vector in \mathbb{R}^2 . Explain why the vectors \vec{v} , $A\vec{v}$, and $A^2\vec{v}$ must be linearly dependent. (Note: do not use any numerical examples in your answer; your reasoning must be valid no matter what \vec{v} is.)

They must be linearly dependent because in P2, only 2 thready independent vectors are needed to form a basis in P? A third vector would always be some linear combination of the first 2 vectors, & therefore would be linearly

(b) Let $\vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Part (a) shows that there are scalars c_0 , c_1 , and c_2 such that $c_0 \vec{v} + c_1 A \vec{v} + c_2 A^2 \vec{v} = \vec{0}$.

Explain why this shows that the matrix $c_0I + c_1A + c_2A^2$ is not invertible.

For a matrix to be invertible, the only vector in its level must be \vec{o}_- If there is any other vector in its level, then it isn't invertible. Because c_0 , c_1 , c_2 are nowzero, the vector $\begin{bmatrix} c_0 \\ c_2 \end{bmatrix}$ is also in the kernel, so therefore $c_0 \cdot \vec{c}_1 \cdot \vec{c}_2 \cdot \vec{c}_3 \cdot \vec{c}_4 \cdot \vec{c}_4 \cdot \vec{c}_4 \cdot \vec{c}_4 \cdot \vec{c}_5 \cdot \vec{c}_5 \cdot \vec{c}_5 \cdot \vec{c}_6 \cdot \vec{c}_6$

(1)

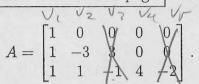
$$A\overrightarrow{V} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A\overrightarrow{V} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 2 & 0 \\ 1 & 0 & 6 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 6 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \rightarrow \overrightarrow{X} = \begin{bmatrix} -6x^3 \\ -x^3 \\ X^3 \end{bmatrix} = X_3 \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

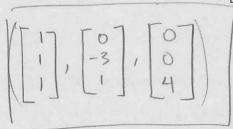
$$\begin{bmatrix} 0 \\ -6 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \end{bmatrix} = 0$$

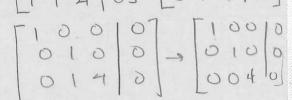
3. (10 points) You do not need to show work on this page. Consider the matrix



 $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -3 & 3 & 0 & 0 \\ 1 & 1 & +1 & 4 & +2 \end{bmatrix}. \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -3 & 0 & 0 & 0 \\ 1 & 1 & 4 & 4 & 4 \end{bmatrix}$

(a) Find a basis for the image of A. Box your answer



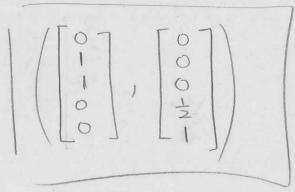


dulina) (() = m

(b) What does the rank-nullity theorem say in this case? (Your answer should be a simple equation of the form 1 + 1 = 2).

(c) Find a basis for the kernel of A. Box your answer

$$V_3 = -V_2 - V_3 + V_2 = 0$$



- 4. (10 points) True/False (circle the correct answer). You do not need to justify your answer.

 Remember that True means always true. If a statement is sometimes true, but sometimes false, mark it False.
 - (a) If A is an invertible matrix then $\ker(A) = \ker(A^{-1})$.

 True

 False

(b) If A is an orthogonal matrix then A^TA is also an orthogonal matrix. We say?

$$A^{-1}=A^{-1}$$
 True False

(c) There exists a subspace V of \mathbb{R}^5 such that $\dim(V) = \dim(V^{\perp})$, where V^{\perp} denotes the orthogonal complement of V.

False)

(d) If V is a two-dimensional subspace of \mathbb{R}^3 (a plane) then there is a basis \mathcal{B} such that the \mathcal{B} -matrix of the linear transformation $T = \operatorname{proj}_V : \mathbb{R}^3 \to \mathbb{R}^3$ has the form $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$

True

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$
True
False

1/= ter (proj v)

(e) If A is a symmetric $n \times n$ matrix and S is an orthogonal $n \times n$ matrix then the matrix $S^{-1}AS$ is symmetric.

AT=A.

True

False

$$(S^{-1}AS)^{T} = S^{T}A^{T}(S^{-1})^{T} = S^{-1}A(S^{T})^{T} = S^{-1}AS$$

$$(S^{-1}AS)^{T} = STA^{T}(S^{-1})^{T} = S^{-1}AS$$

- 5. (10 points) The following statements are the possible answers to questions (a) and (b) below.
 - (A) The linear system $A\vec{x} = \vec{0}$ has a unique solution \checkmark
 - (B) $\dim(\operatorname{im} A) + \dim(\ker A) = n$
 - (C) rank(A) = n
 - (D) A is upper triangular
 - (E) The column vectors of A span \mathbb{R}^n \checkmark
 - (F) $A^T A = I_n$
 - (G) $rank(A) = rank(A^T)$
 - (H) $A^n = 0$
 - (I) $||A\vec{x}|| = ||\vec{x}||$ for all \vec{x} in \mathbb{R}^n
 - (a) Which of the statements above implies that the $n \times n$ matrix A is invertible? There are five correct answers.











(b) Suppose that A is an invertible $n \times n$ matrix. Which of the properties above does A satisfy? There are five correct answers.







