Full Name Christopher Kha

Redmond McNamara T GEOLOG

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2A	Redmond McNamara	T	GEOLOGY 6704
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- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
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Page:	1	2	3	4	5	Total
Points:	10	10	10	10	10	50
Score:						

You may use this page for scratch work.

Page 0

1. (10 points) (a) Which of the following is the QR-factorization of the matrix

$$A = \begin{bmatrix} 1 & 1 & 9 \\ 1 & 2 & -4 \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$
?

Circle your answer

$$\begin{bmatrix}
1/2 & -1/2 & 1/2 \\
1/2 & 1/2 & -1/2 \\
1/2 & 1/2 & 1/2 \\
1/2 & -1/2 & -1/2
\end{bmatrix}
\begin{bmatrix}
2 & 3 & 5 \\
0 & 1 & -6 \\
0 & 0 & 7
\end{bmatrix}$$

$$\begin{bmatrix}
1/\sqrt{2} & 0 & -1/2 \\
0 & 1/\sqrt{2} & -1/2 \\
1/\sqrt{2} & 0 & 1/2 \\
0 & 1/\sqrt{2} & 1/2
\end{bmatrix}
\begin{bmatrix}
2 & 0 & 12 \\
0 & 12 & 18 \\
0 & 0 & 6\sqrt{2}
\end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} & 0 & -1/2 \\ 0 & 1/\sqrt{2} & -1/2 \\ 1/\sqrt{2} & 0 & 1/2 \\ 0 & 1/\sqrt{2} & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 12 \\ 0 & 12 & 18 \\ 0 & 0 & 6\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & 44 \\ 1 & -4 & 22 \\ 1 & -4 & 29 \\ 1 & -5 & 37 \end{bmatrix} \begin{bmatrix} 1 & 6 & 10 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & 44 \\ 1 & -4 & 22 \\ 1 & -4 & 29 \\ 1 & -5 & 37 \end{bmatrix} \begin{bmatrix} 1 & 6 & 10 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

(b) Write down the result of the Gram-Schmidt process on the vectors

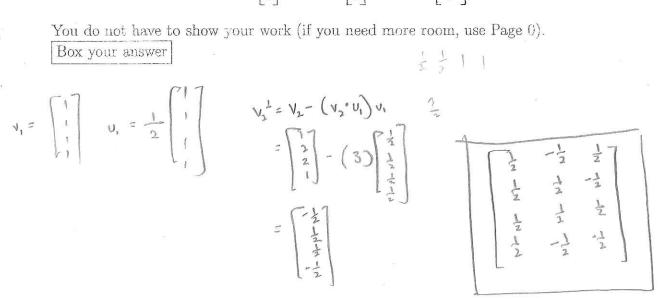
$$ec{v_1} = egin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad ec{v_2} = egin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \quad ec{v_3} = egin{bmatrix} 9 \\ -4 \\ 3 \\ 2 \end{bmatrix}.$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad v_2 = \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$V_{2}^{1} = V_{2} - (V_{2}, V_{1}) V_{1}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} - (3) \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$



2. (10 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} \qquad A^2 = \begin{bmatrix} 0 & 4 \\ -4 & 8 \end{bmatrix}$$

(a) Suppose that \vec{v} is any nonzero vector in \mathbb{R}^2 . Explain why the vectors \vec{v} , $A\vec{v}$, and $A^2\vec{v}$ must be linearly dependent. (Note: do not use any numerical examples in your answer; your reasoning must be valid no matter what \vec{v} is.)

reasoning must be valid no matter what
$$\vec{v}$$
 is.)

$$v = \begin{bmatrix} a \\ b \end{bmatrix} \qquad A\vec{v} = a \begin{bmatrix} 1 \\ -1 \end{bmatrix} + b \begin{bmatrix} 3 \end{bmatrix} = \begin{bmatrix} a+b \\ 3b-a \end{bmatrix} \qquad \begin{bmatrix} a+b \\ 3b-a \end{bmatrix} = \begin{bmatrix} a+b \\ b \end{bmatrix} = \begin{bmatrix} a+$$

$$c_0 \vec{v} + c_1 A \vec{v} + c_2 A^2 \vec{v} = \vec{0}. \tag{1}$$

Explain why this shows that the matrix $c_0I + c_1A + c_2A^2$ is not invertible. In order for the matrix to be invertible, the columns must be livearly independent. However, since cov+c, Av+c2A2v=0. refers to a linear combination of the first columns of I, A and A2 and this combination is linearly dependent, then to matrix coll & C. A 462 A2 must also be limearly dependent and this not inventible

(c) Let $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Find scalars c_0 , c_1 , and c_2 such that (1) holds. Box your answer

3. (10 points) You do not need to show work on this page. Consider the matrix

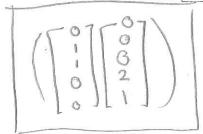
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -2 & 2 & -2 & 4 \end{bmatrix}.$$

(a) Find a basis for the image of A. Box your answer



(b) What does the rank-nullity theorem say in this case? (Your answer should be a simple equation of the form 1+1=2).

(c) Find a basis for the kernel of A. Box your answer



- 4. (10 points) True/False (circle the correct answer). You do not need to justify your answer. Remember that True means always true. If a statement is sometimes true, but sometimes false, mark it False.
 - (a) There exists a subspace V of \mathbb{R}^5 such that $\dim(V) = \dim(V^{\perp})$, where V^{\perp} denotes the orthogonal complement of V.

(b) If A is an invertible matrix then $ker(A) = ker(A^{-1})$.

- (c) If A is an orthogonal matrix then AA^T is also an orthogonal matrix.

 True

 False
- (d) If A is a symmetric $n \times n$ matrix and S is an <u>orthogonal</u> $n \times n$ matrix then the matrix $S^{-1}AS$ is symmetric.

$$(S^-AS)^T = S^TA^TS^{-T} = S^-AS$$

(e) If V is a two-dimensional subspace of \mathbb{R}^3 (a plane) then there is a basis \mathcal{B} such that the \mathcal{B} -matrix of the linear transformation $T = \operatorname{proj}_V : \mathbb{R}^3 \to \mathbb{R}^3$ has the form

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

5. (10 points) The following statements are the possible answers to questions (a) and (b) below.

- (A) The column vectors of A span \mathbb{R}^{n}
- (B) $A^T A = I_n$
- (C) $rank(A) = rank(A^T)$
- (D) $A^n = 0$
- (E) $||A\vec{x}|| = ||\vec{x}||$ for all \vec{x} in $\mathbb{R}^n \setminus$

(F) The linear system $A\vec{x} = \vec{0}$ has a unique solution \sim

- (G) $\dim(\operatorname{im} A) + \dim(\ker A) = n$
- (H) $\operatorname{rank}(A) = n$
- (I) A is upper triangular

(a) Which of the statements above implies that the $n \times n$ matrix A is invertible? There are five correct answers.

B

F

H

A

E

(b) Suppose that A is an invertible $n \times n$ matrix. Which of the properties above does A satisfy? There are five correct answers.

G

F

B

A

H