MATH 33A Final Exam Spring 2019

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- Fill out your name, section letter, and UID above.
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- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

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Points:	10	8	10	10	12	10	20	20	100
Score:									

1. (5 points) Suppose that A is a 5×4 matrix of the form

$$A = \begin{bmatrix} | & | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \\ | & | & | & | \end{bmatrix}.$$

Given that the vector $\begin{bmatrix} 7 \\ 3 \\ -2 \\ 8 \end{bmatrix}$ is in the kernel of A, write $\vec{v}_4$ as a linear combination of the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$. Box your answer.

$$\begin{bmatrix} | & | & | & | \\ \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \\ | & | & | & | \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ -2 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7\vec{v}_1 + 3\vec{v}_2 - 2\vec{v}_3 + 8\vec{v}_4 = \vec{0}$$

$$8\vec{v}_4 = -7\vec{v}_1 - 3\vec{v}_2 + 2\vec{v}_3$$

$$\boxed{\vec{v}_4 = -\frac{7}{8}\vec{v}_1 - \frac{3}{8}\vec{v}_2 + \frac{1}{4}\vec{v}_3}$$

2. (5 points) Suppose that the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the orthogonal projection onto the plane $3x_1 + x_2 - 2x_3 = 0$. Find a basis for $\text{im}(T)$ and a basis for $\ker(T)$. Box your answers.

$$\begin{bmatrix} 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \vec{0}$$

$x_1 = t$

$x_2 = 2x_3 - 3x_1 = 2s - 3t$

$x_3 = s$

$$= t\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$\boxed{\text{vectors } \left(\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\right) \text{ form a basis of } \text{im}(T)}$

$\boxed{\text{vector } \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} \text{ forms a basis for } \ker(T)}$

3. (8 points) For each of the 2×2 matrices A below, there is an invertible matrix S such that $B = S^{-1}AS$ is either a diagonal matrix $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ or a rotation-scaling matrix $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. Find B in each case (you do not have to find S).

(a) $A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 3-\lambda & 1 \\ -2 & 5-\lambda \end{bmatrix}$

$(3-\lambda)(5-\lambda) + 2 = 0$

$\lambda^2 - 8\lambda + 15 + 2 = 0$

$\lambda^2 - 8\lambda + 17 = 0$

$\lambda = \frac{8 \pm \sqrt{64 - 4(17)}}{2} = \frac{8 \pm \sqrt{-4}}{2} = \overset{a}{4} \pm \overset{b}{i}$

$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = S^{-1} A S$

$\begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}^{-1} A \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$

$\lambda = 4+i$ $\quad \ker \begin{bmatrix} -1-i & 1 \\ -2 & 1-i \end{bmatrix} \rightarrow \ker \begin{bmatrix} -1-i & 1 \\ -1-i & 1 \end{bmatrix} \rightarrow \ker \begin{bmatrix} -1-i & 1 \\ 0 & 0 \end{bmatrix}$

$-2 = (-1)(1-i)(1+i)$ $\quad (-1)(1+i)$

$= \operatorname{Span} \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$

$= \frac{1}{-2} \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$

$= \frac{1}{-2} \begin{bmatrix} 8 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} -8 & 2 \\ -2 & -8 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}$

$$\boxed{B = \begin{bmatrix} 4 & -1 \\ 1 & 4 \end{bmatrix}}$$

$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + i \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 5-\lambda & -4 \\ 2 & -1-\lambda \end{bmatrix}$

$(5-\lambda)(-1-\lambda) + 8 = 0$

$\lambda^2 - 4\lambda - 5 + 8 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\lambda = 1, 3$

$$\boxed{B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}}$$

4. (10 points) Let $T(\vec{x}) = A\vec{x}$ be the linear transformation with matrix

$$A = \begin{bmatrix} 4 & -3 & 2 \\ -2 & 3 & 2 \\ 5 & -5 & 4 \end{bmatrix}.$$

$$\begin{tikzcd} \cdot \arrow[r, "A"] & \cdot \\ \cdot \arrow[u, "S"] \arrow[r, "B"'] & \cdot \arrow[u, "S"'] \end{tikzcd}$$

(a) Let $\mathcal{B}$ be the basis of $\mathbb{R}^3$ given by

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}.$$

$B_x = S^{-1} A S x$

Find the matrix of T in the basis $\mathcal{B}$.
(Another way to say this is: find a matrix B such that $[T(\vec{x})]_{\mathcal{B}} = B[\vec{x}]_{\mathcal{B}}$.)

$B = S^{-1} A S$ $\quad S = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$S^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] (I - S)^{*} - 1$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$

$\underbrace{\qquad\qquad}_{S^{-1}}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 4 & -3 & 2 \\ -2 & 3 & 2 \\ 5 & -5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4 & -3 & 2 \\ 5 & -5 & 4 \\ 6 & -6 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$\boxed{B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}}$

(b) Find the eigenvalues of A, repeating any eigenvalues according to their algebraic multiplicities. (So if $\lambda_2 = 17$ has algebraic multiplicity 2, list it twice.)
Hint: use your answer from part (a), which should be nice enough that you don't need to compute a determinant for this part.

upper tri

$\boxed{\lambda_1 = 1, \quad \lambda_2 = 4, \quad \lambda_3 = 6}$

5. (10 points) Make sure to fully justify your answers on this page.

Suppose that A is a symmetric $n \times n$ matrix (this is an assumption for both (a) and (b) below).

(a) Show that $A\vec{v} \cdot \vec{w} = \vec{v} \cdot A\vec{w}$ for any two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$.
Hint: remember that another way to write the dot product is $\vec{x} \cdot \vec{y} = \vec{x}^T \vec{y}$.

$$\begin{aligned} A\vec{v} \cdot \vec{w} &= (A\vec{v})^T \vec{w} \\ &= \vec{v}^T A^T \vec{w} \\ &= \vec{v}^T A \vec{w} \quad \leftarrow A \text{ symmetric} \\ &= \vec{v} \cdot (A\vec{w}) \quad \checkmark \end{aligned}$$

(b) Suppose that $\vec{v}$ and $\vec{w}$ are eigenvectors of A with eigenvalues λ and μ, respectively. Show that if $\lambda \neq \mu$ then $\vec{v}$ is orthogonal to $\vec{w}$.

$A\vec{v} = \lambda \vec{v}$ $\qquad$ $A\vec{w} = \mu \vec{w}$

$A\vec{v} \cdot \vec{w} = \vec{v} \cdot A\vec{w}$

$\lambda(\vec{v} \cdot \vec{w}) = \mu(\vec{v} \cdot \vec{w})$

If $\lambda \neq \mu$, the only solution to this system would be if $\vec{v} \cdot \vec{w} = 0$

so $\vec{v}$ and $\vec{w}$ are orthogonal

6. (12 points) Define

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues of A and state the algebraic multiplicity of each eigenvalue. (No justification needed for this part.)

$\lambda_1 = 1$ alg. mult. = 2

$\lambda_2 = 0$ alg. mult. = 2

(b) Find a basis for each eigenspace.

$$\lambda_1 = 1 \quad \ker(A - I) = \ker \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \ker \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{span} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{\left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right) \text{ is a basis for } E_1}$$

$$\lambda_2 = 0 \quad \ker(A) = \ker \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \ker \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \text{span} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) \text{ is a basis for } E_2}$$

(c) State the geometric multiplicity of each eigenvalue.

$\lambda_1 = 1$ geo mult = 1

$\lambda_2 = 0$ geo mult = 2

(d) Is A diagonalizable? Justify your answer.

A is not diagonalizable because the geometric multiplicities do not add up to 4.

$D = S^{-1} A S$

$A = S D S^{-1}$

7. (10 points) For this question, it may be helpful to know that

$$\underbrace{\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}}_{A} = \underbrace{\begin{array}{c} \begin{array}{ccc} v_2 & v_3 & v_1 \end{array} \\ \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix} \end{array}}_{S} \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}}_{S^{-1}}.$$

Find a singular value decomposition $A = U\Sigma V^T$ of the matrix

$$\underset{2\times 3}{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Write your answer in the form $\boxed{U = \text{something}, \Sigma = \text{something}, V = \text{something}}$.

2×2 $\quad$ 3×3

$$M = A^TA = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$\lambda_1 = 3 \quad \lambda_2 = 2 \quad \lambda_3 = 0$

$\sigma_1 = \sqrt{3} \quad \sigma_2 = \sqrt{2}$

$$\vec{v}_1 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad \vec{v}_3 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$\vec{u}_1 = \frac{1}{\sigma_1} A\vec{v}_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 0 \\ \frac{3}{\sqrt{3}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\vec{u}_2 = \frac{1}{\sigma_2} A\vec{v}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} -\frac{2}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\boxed{U = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} \quad V = \begin{bmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix}}$$

Check

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\sqrt{2} & 0 \\ \sqrt{3} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\sqrt{2} & 0 \\ \sqrt{3} & 0 & 0 \end{bmatrix}\underbrace{\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}}_{V^T} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

8. (20 points) True/False (circle the correct answer). You do not need to justify your answer.

Remember that True means **always true**. If a statement is sometimes true, but sometimes false, mark it False.

(a) Every 5×5 matrix has a real eigenvalue.

True   False

(b) If A is an $n \times n$ matrix such that $\text{im}(A) = \{\vec{0}\}$ then A is invertible.

True   False

$(UAU^T)^T = U^{T^T} A^T U^T$
$= UAU^T$

(c) If A is symmetric and U is orthogonal then UAU^T is symmetric.

True   False

(d) If A is a symmetric matrix and λ is one of its eigenvalues, then λ is also one of the singular values of A.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\lambda_1 = 1$ $\lambda_2 = -1$ $\lambda = 1$

True   False

(e) The matrices $\begin{bmatrix} 8 & 6 & 7 \\ 5 & 3 & 0 \\ 9 & 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 4 & 8 & 1 \\ 5 & 1 & 6 \\ 23 & 4 & 2 \end{bmatrix}$ are similar.

$tr = 12$ $tr = 7$

True   False

(f) If $\ker(A^3) = \ker(A^4)$ and $\vec{v}$ is a vector in $\ker(A^5)$, then $\vec{v}$ is automatically in $\ker(A^4)$.

$A(AAAA\vec{v}) = 0$

True   False

(g) If A is a 4×7 matrix then $\text{rank}(A) + \dim(\ker(A)) = 4$.

True   False

(h) If A is a symmetric $n \times n$ matrix such that $A^n = 0$ then $A = 0$.

True   False

(i) If A is an upper triangular $n \times n$ matrix such that $A^n = 0$ then $A = 0$.

True   False

(j) If A is an $n \times n$ matrix such that $\|A\vec{x}\| = \|\vec{x}\|$ for all $\vec{x}$ in $\mathbb{R}^n$ then A is an orthogonal matrix.

True   False

9. (20 points) Fill in the blanks with the matrices below (just write A, B, etc. in each blank). You may use some items more than once, and there are some items that you will not use at all. **If none of the matrices on this list makes the statement true, write NONE.**

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad F = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad K = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(a) Suppose that $M\vec{x} = \vec{b}$ is a linear system with 3 equations and 3 unknowns. If the system has a unique solution then the reduced row echelon form of M equals __J__.

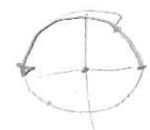
(b) Suppose that R is a 3×2 matrix and that S is a 2×3 matrix. If $\text{im}(R) = \ker(S)$ then $SR =$ __A__.

R S

$\begin{bmatrix} 3\times2 \end{bmatrix} \begin{bmatrix} 2\times3 \end{bmatrix}$

$SR =$

$S(Rx) = 0$



(c) If M is a 2×2 rotation by $\pi/6$ radians counterclockwise, then $M^{18} =$ __F__.

$\left(\frac{\pi}{6}\right) 18 = 3\pi$

(d) $M =$ __B__ is a 2×2 matrix such that $\text{im}(M) = \ker(M)$.

(e) $M =$ __None__ is a 3×3 matrix such that $\text{im}(M) = \ker(M)$.

(f) $\lambda = 0$ is the only eigenvalue of __H__. Furthermore, the algebraic multiplicity of λ is 3 and the geometric multiplicity of λ is 1.

(g) tr(__E__) = 1.

2 correct

(h) $M =$ __H, K__ is a matrix with rank 2 whose kernel is not $\{\vec{0}\}$.

(i) If M is an orthogonal 3×3 matrix then $M^T M = M M^T =$ __J__.

(j) __D__ has complex (non-real) eigenvalues.

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

You may use this page for scratch work.