Name: ______________________________ Student ID: ____________ Section: 1___

Prof. Alan J. Laub April 29, 2011

Math 33A/1 – MIDTERM EXAMINATION
Spring 2011

Instructions:

(a) The exam is closed-book (except for one page of notes) and will last 50 minutes. No calculators, cell phones, or other electronic devices are allowed at any time.

(b) Notation will conform as closely as possible to the standard notation used in the lectures, not the textbook.

(c) Do all 4 problems. Each problem is worth 25 points. Hand in the exam with work shown where appropriate. Some partial credit may be assigned if warranted. Label clearly the problem number and the material you wish to be graded.

(d) You must enter your discussion section above.

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1. (a) Consider the tridiagonal matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. Perform Gaussian elimination on A to reduce it to upper triangular form U.

   (b) Factor A in the form LU where L is unit lower triangular.

   (c) What is $\det(A)$?

2. Write all solutions of the equations $Ax = b$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$ in the form $x_p + x_n$ where x_p denotes the particular solution which solves $Ax_p = b$ and the special solutions x_n solve $Ax_n = 0$.

3. Consider the vector space $\mathbb{R}^{n\times n}$ over $\mathbb{R}$ and let $\mathcal{U}$ be the subset of upper triangular $n \times n$ matrices (i.e., matrices A for which $a_{ij} = 0$ for $i > j$) and let $\mathcal{L}$ be the subset of strictly lower triangular $n \times n$ matrices (i.e., matrices A for which $a_{ij} = 0$ for $i \leq j$).

   (a) Show that $\mathcal{U}$ and $\mathcal{L}$ are subspaces of $\mathbb{R}^{n\times n}$.

   (b) Show that $\mathcal{U} \oplus \mathcal{L} = \mathbb{R}^{n\times n}$.

4. Let

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 4}.$$

(a) Put A in reduced row echelon form (rref).

(b) What is rank(A)? What is rank(A^T)?

(c) Find a basis for $\mathcal{N}(A)$ (the nullspace of A) from rref(A).

(d) Find a basis for $\mathcal{C}(A)$ (the columnspace of A) from rref(A).

(e) Find a basis for $\mathcal{C}(A^T)$ from rref(A).

(f) The matrix $E = \begin{bmatrix} \frac{5}{2} & -2 & 0 \\ -1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ puts A in rref. Using this E, find a basis for $\mathcal{N}(A^T)$.